Adaptive Proxy-Based Robust Control Integrated With Nonlinear Disturbance Observer for Pneumatic Muscle Actuators

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Abstract—In pneumatic muscle actuators (PMAs)-driven robotic applications, there might exist unpredictable shocks which lead to the sudden change of desired trajectories and large tracking errors. This is dangerous for physical systems. In this article, we propose a novel adaptive proxy-based robust controller (APRC) for PMAs, which is effective in realizing a damped response and regulating the behaviors of the PMA via a virtual proxy. Moreover, the integration of the APRC and the nonlinear disturbance observer further handles the system uncertainties/disturbances and improves the system robustness. According to the Lyapunov theorem, the tracking states of the closed-loop PMA control system are proven to be globally uniformly ultimately bounded through two motion phases. Extensive experiments are conducted to verify the superior performance of our approach, in multiple tracking scenarios.

Index Terms—Adaptive proxy-based robust control, pneumatic muscle actuator, two-phase stability analysis.

I. INTRODUCTION

Due to the attractive characteristics, i.e., high power/weight ratio, no mechanical parts, low cost, etc. [1], the pneumatic muscle actuator (PMA) has been widely used in a variety of fields, especially exoskeletons that are effective in power augmentation and rehabilitation training [2]–[4]. Its driving force is converted from the air pressure of the inner bladder, which has the features of nonlinearity, hysteresis, and time-varying parameters [5], making its modeling and control very challenging. Different control strategies have been proposed for the PMA, including proportional-integral-derivative (PID)-based control [6], nonlinear model predictive control [7], [8], sliding-mode control (SMC) [9], fuzzy control [10], adaptive control [11], dynamic surface control [12], etc. Unfortunately, an accurate mathematic model of the PMA is very difficult to obtain in practice, which causes difficulties in precise control. Meanwhile, the traditional PID control, a typical model-free strategy, works in the position control of the PMA. However, some significant issues should be taken into account. First, the high-gain PID controller may cause oscillation and can hardly realize satisfactory performance in the physical PMA applications, due to the slow response of the PMA and limited sampling rate. Second, the PMA is widely used in the field of robot actuation and industry, in which the load, running amplitude, and frequency may change within a certain task. The traditional PID controller with a set of fixed control parameters may not meet the requirements of these applications. Next, from a theoretical viewpoint, it is difficult to theoretically prove the stability of the closed-loop system when no theoretical model is involved. Thus, there is still a strong demand for robust PMA control.

In robotic applications, the idea of using a proxy is common because a proxy enables robots to track the reference with a damped response to unexpected impacts, which results in the improvement of the system security and performance [13]. However, the physical proxy requires a light-weight and compact mechanism that leads to difficulties for designation. The virtual proxy is a remedy to fulfill the requirement of robot control. A typical strategy called proxy-based sliding-mode control (PSMC) [14], which assumes that a zero-quality virtual proxy exists between the controlled object and the desired trajectory, is significantly a model-free strategy. Damme et al. [15] presented
a PSMC for a two-degree-of-freedom planar manipulator actuated by pleated pneumatic artificial muscles, and such a strategy of position control was developed for piezoelectric-actuated nanopositioning stages in [16]. Another approach supposed that there was a free space around the proxy for the impedance control of a cable-driven system [17]. However, most proxy-based strategies lack stability analysis or depend on a strong conjecture (e.g., see Conjecture 1 in [14]). Therefore, this kind of strategies demands further investigation to establish a sound theoretical foundation.

The robustness of the control strategy is another significant issue for robotic systems. Although the proxy-based strategies have been used in various applications, most of them rarely consider the improvement of system robustness. Nonlinear disturbance observer (NDO)-based control is a common method for improving control performance. The basic idea is to estimate the disturbances/uncertainties from measurable variables before a control action is taken. Consequently, the influence of the disturbances/uncertainties can be suppressed, and the system becomes more robust [18]–[20]. Multiple NDO-based control strategies have been proposed to compensate for the influence of disturbances/uncertainties [21]–[25]. However, to our best knowledge, there are very few researches on the proxy-based control strategy integrated with NDO. This may be due to two challenges. First, most of the proxy-based strategies are model-free control approaches, whereas a typical NDO-based controller requires a mathematical model of the control system. Therefore, the integration of proxy-based strategy and NDO is not straightforward. Second, a more rigorous analysis is needed to guarantee the stability of the system, which should not be based on a strong conjecture.

This article proposes an adaptive proxy-based robust control integrated with nonlinear disturbance observer for the position control of PMAs. Our main contributions are as follows:

1) The proposed adaptive proxy-based robust control extends PSMC from a model-free strategy to a model-based strategy by defining the motion behaviors of the proxy. Accompanied by a nonlinear disturbance observer, the proposed control method retains the original characteristics of smooth and damped motions and greatly improves the robustness of the algorithm.

2) The proposed controller ensures the global stability of the closed-loop system through two stages, in which the controlled object tracks the proxy, and the proxy tracks the reference trajectory, simultaneously. Furthermore, this article elaborately studies the case when the proxy is not zero and finds that the nonzero proxy mass is capable of regulating the behaviors of the controlled object.

3) Real-world experiments are conducted based on a physical PMA platform for validating the effectiveness of the proposed controller, and the results present better tracking accuracy and robustness under various reference trajectories.

Fig. 1. PMA and its three-element model.

II. THREE-ELEMENT MODEL OF THE PMA

The generalized three-element model of the PMA is shown in Fig. 1 [27]. The contractile length varies with the air pressure of inner bladder. The dynamics of the PMA is

\[
\begin{align*}
\dot{x} + b(P)x + k(P)x = f(P) - mg \\
b_i(P) = b_{i0} + b_{i1}P \quad \text{(inflation)} \\
b_d(P) = b_{d0} + b_{d1}P \quad \text{(deflation)} \\
k(P) = k_0 + k_1P \\
f(P) = f_0 + f_1P
\end{align*}
\]

where \(m, x, P\) are the mass of load, the contractile length of PMA, and the air pressure, respectively. \(b_i(P), f(P), k(P)\) are the damping coefficient, the contractile force, and the spring coefficient, respectively.

Let \(\tau(t)\) denote the sum of unmodeled uncertainties, including unmodeled dynamics, friction, inaccurate parameters, and changing loads. The dynamics of the PMA can be rewritten as a typical second-order nonlinear model

\[
\begin{align*}
\dot{x} &= f(x, \dot{x}) + b(x, \dot{x})u + \tau(t) \\
\dot{f}(x, \dot{x}) &= \frac{1}{m} (f_0 - mg - b_0\dot{x} - k_0x) \\
b(x, \dot{x}) &= \frac{1}{m} (f_1 - b_1\dot{x} - k_1x)
\end{align*}
\]

where \(u\) is the air pressure, and \(f(x, \dot{x})\) and \(b(x, \dot{x})\) are nonlinear terms related to the system states.
connected to the physical actuator, is presented. Before intro-
ducing the controller, an imaginary object called “proxy,” assumed to be
PMA to track the desired trajectory. In our proxy-based robust
controller, the dynamics can be written as
\[ \dot{x}_d = c_1(x_d - \hat{x}) + c_2(x_d - x), \]
where \( c_1 \) and \( c_2 \) are positive constants, \( x_d \) is the desired trajectory,
and \( x \) is the actual position. Hence, we generate the control signal of proxy
\( u_r \) between the desired trajectory and the proxy, i.e.
\[ u_r = \hat{\Gamma} \cdot \text{sgn}(S_p) \]
where \( \text{sgn}(S_p) \) is the signum function. \( \hat{\Gamma} \) is the adaptive gain of
the sliding surface \( S_p \), and the corresponding optimal constant of \( \hat{\Gamma} \) is \( \Gamma^* \).

The adaptive law is described as
\[ \dot{\hat{\Gamma}} = \begin{cases} \gamma |S_p|, & |S_p| \geq \delta \\ 0, & |S_p| < \delta \end{cases}, \hat{\Gamma}(0) = 0 \]
where \( \gamma \) is a positive constant that regulates the adaptive rate.
\( \delta \) is a boundary layer. When the system achieves a steady state,
\( |S_p| \) is small enough, so that \( \hat{\Gamma} \) will reach an upper bound instead
of monotonically increasing.

Remarkably, the proxy is affected by \( u_r \) and \( u_l \), simultane-
ously, as shown in Fig. 2, and they are not force signals in the
traditional sense. Hence, we cannot directly use Newton’s law
to establish the relationship between the motion behaviors of the proxy
and \( u_r \) and \( u_l \). Besides, it is necessary to define such property
to ensure the realization of tracking and the system’s stability.

Similar to Newton’s law, we define the behavior of the proxy
under the effects of \( u_r \) and \( u_l \). Let \( m_p > 0 \) be the so-called
proxy mass. Then
\[ m_p \dot{x}_p = -u_r + u_l. \]

The effect of \( -u_r + u_l \) is similar to the resultant force on
the proxy while \( m_p \dot{x}_p \) can be seen as the motion principle of
the proxy. Note that this property can be arbitrarily defined
according to the specific situation, as long as the stability of the
closed-loop system can be ensured.

Combining (4), (7), (9), and (11), the trajectory of the proxy
is presented as
\[ \dot{x}_p = \frac{1}{m_p} \left[ \hat{\Gamma} \text{sgn}(S_p) - K_p(x_p - x) - K_i \int (x_p - x) \right], \]
\[ -K_d(\dot{x}_p - \hat{x}) + \dot{x}_d + c_1(\dot{x}_d - \hat{x}) + c_2(x_d - x). \]

Once \( x_p \) is determined, the control signal of the PMA can then
be computed from (4), (8), and (12).

For the convenience of presentation, we first define \( K_m = \text{diag}\{K_i, c_2, \omega, K_d\} \) with \( \omega = K_p c_1 - K_i - K_d c_2. \)
\textbf{Theorem 1:} The norm of tracking error between the proxy states $X_p = [\int x_p dt, x_p, \dot{x}_p]^T$ and the system states $X = [\int x dt, x, \dot{x}]^T$ is uniformly ultimately bounded, and a sliding motion on the surface (4) can be guaranteed when the APRC satisfies

$$m_p > 0, \lambda(K_c) > 0, \Gamma^* \geq \lambda_2(K_p + K_i + K_d), \omega > 0$$

where $\lambda(\cdot)$ and $\lambda_{\text{min}}(\cdot)$ denote the eigenvalues and the minimum eigenvalue of the matrix, respectively.

Proof: Due to $\lambda(K_c) > 0$, a Lyapunov candidate is defined as

$$V = V_1 + V_2 + V_3 > 0$$

with

$$V_1 = \frac{1}{2} m_p \dot{S}_p^2 + \frac{1}{2} S_q^2$$

$$V_2 = \frac{1}{2} [e_p \dot{e}_p] K_c [e_p]$$

$$V_3 = \frac{1}{2} \dot{\Gamma}^2$$

where $e_p = \int (x_p - x) dt$ and $\dot{\Gamma} = \Gamma - \Gamma^*$.

From (7)–(11), it follows that

$$m_p \dot{S}_p = -\dot{\Gamma} \text{sgn}(S_p) + K_p \dot{e}_p + K_i e_p + K_d \dot{e}_p.$$  

(16)

Integrating (10)–(18), the derivatives of $V_1, V_2, V_3$ are

$$\dot{V}_1 = S_p(-\dot{\Gamma} \text{sgn}(S_p) + K_p \dot{e}_p + K_i e_p + K_d \dot{e}_p)$$

$$+ S_q(-K_p \dot{e}_p - K_i e_p - K_d \dot{e}_p - \tau)$$

$$= -\dot{\Gamma} |S_p| - \tau S_q + (K_p \dot{e}_p + K_i e_p + K_d \dot{e}_p)(S_p - S_q)$$

$$= -\dot{\Gamma} |S_p| - \tau S_q - K_d \dot{e}_p^2 - K_p \dot{e}_p - K_i e_p + K_d \dot{e}_p$$

$$= -\Gamma^* |S_p| - \tau S_q - K_d \dot{e}_p^2 - K_p \dot{e}_p - K_i e_p + K_d \dot{e}_p$$

(19)

$$V_2 = (K_p + K_d \dot{e}_p) \dot{e}_p + (K_p + K_d \dot{e}_p) \dot{e}_p$$

$$+ (K_i + K_d \dot{e}_p) \dot{e}_p + (K_i + K_d \dot{e}_p) \dot{e}_p$$

$$= \frac{1}{2} \dot{\Gamma} \dot{\Gamma} = \Gamma |S_p| = \Gamma |S_p| - \Gamma^* |S_p|.$$  

(21)

Then, it follows that

$$\dot{V}_1 + \dot{V}_2 = -\dot{\Gamma} |S_p| - \tau S_q - K_d \dot{e}_p^2 - \tau e_p^2 - K_i \dot{e}_p^2.$$  

(22)

Note that

$$\Gamma^* \geq \frac{(K_p + K_i + K_d)(1 + c_1 + c_2)}{\min(K_i c_2, \omega, K_d)} \varepsilon \geq \varepsilon.$$  

(23)

From (17)–(23), we have

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$$

$$= -\Gamma^* |S_p| - \tau S_q - K_d \dot{e}_p^2 - \tau e_p^2 - K_i \dot{e}_p^2$$

$$\leq -\varepsilon |S_p| + (K_p \dot{e}_p + K_i e_p + K_d \dot{e}_p)$$

$$- \tau S_q - K_d \dot{e}_p^2 - \tau e_p^2 - K_i \dot{e}_p^2$$

$$\leq \varepsilon |1 + c_1 + c_2| |e_p| - \lambda_{\text{min}}(K_c) |e_p|^2$$

$$= -\|e_p\|^2 - \lambda_{\text{min}}(K_m) |e_p|^2 - \varepsilon |1 + c_1 + c_2|.$$  

(24)

where $e_p = X_p - X = [e_p, \dot{e}_p, \dot{e}_p]^T$. It is easy to see that after a sufficiently long time

$$\|e_p\| \leq \lambda_2.$$  

(25)

As a result, $\|e_p\|$ is uniformly ultimately bounded.

Define a new Lyapunov candidate as

$$V_p = \frac{1}{2} m_p \dot{S}_p^2 + \frac{1}{2} \dot{\Gamma}^2.$$  

(26)

It follows from (16) that

$$\dot{V}_p = m_p \dot{S}_p \dot{S}_p + \frac{1}{\gamma} \dot{\Gamma} \dot{\Gamma}$$

$$= -\Gamma^* |S_p| + (K_p \dot{e}_p + K_i e_p + K_d \dot{e}_p)$$

$$\leq -\Gamma^* |S_p| + \lambda_2(K_p + K_i + K_d) S_p$$

$$\leq 0.$$  

(27)

When $\|e_p\|$ is uniformly ultimately bounded, the achievement of a sliding motion on the surface (4) is guaranteed. This completes the proof.

Remark 2: The stability analysis of the system has two motion phases. First, the norm of the tracking error between the proxy states $X_p$ and the system states $X$ is uniformly ultimately bounded. This indicates that the system states converge to the proxy states. Then, the achievement of sliding motion on the surface (4) means that the proxy tracks the reference trajectory, theoretically. In summary, the system states are capable of indirectly tracking the reference, and the stability of the closed-loop system is guaranteed.

Corollary 1: If inequality (25) holds, and initially $x_p = x_d$, then, as the proxy mass $m_p$ increases, $S_q$ will gradually approach a bound associated with the upper bound of the lumped disturbances.

$$\lim_{m_p \rightarrow \infty} \|S_q\| \leq \lambda_2(c_1 + c_2 + 1).$$  

(28)

Proof: From (16), it follows that

$$|\dot{S}_p| = \frac{1}{m_p} |\dot{\Gamma} \text{sgn}(S_p) + K_p \dot{e}_p + K_i e_p + K_d \dot{e}_p|.$$  

(29)
Since the system is globally uniformly ultimately bounded and a limited $\Gamma^*$, we have
\[
\lim_{m_p \to \infty} |\dot{S}_p| = 0. \tag{30}
\]
The proxy mass $m_p$ is a fixed value in each experiment. Let $t_f$ be the finite duration of the experiment. Then,
\[
S_p = \int_0^{t_f} \dot{S}_p \, dt + v \tag{31}
\]
where $v$ is the initial value of $x_d - x_p$, which equals zero. Hence, it follows that
\[
|S_p| = |\int_0^{t_f} \dot{S}_p \, dt| \leq \int_0^{t_f} |\dot{S}_p| \, dt. \tag{32}
\]
Combining (30) and (32), we can obtain
\[
\lim_{m_p \to \infty} |S_p| = 0. \tag{33}
\]
Considering (18) and (25), after a sufficiently long time
\[
|S_q| \leq |S_p| + |e_p| + c_1 \dot{e}_p + c_2 \ddot{e}_p| \leq |S_p| + \lambda_2 (c_1 + c_2 + 1). \tag{34}
\]
Finally,
\[
\lim_{m_p \to \infty} |S_q| \leq \lambda_2 (c_1 + c_2 + 1). \tag{35}
\]
According to the above results, when the proxy mass $m_p$ approaches positive infinity, $S_q$ will approach a bound associated with the upper bound of the lumped disturbances. This means that $m_p$ can be used to regulate the behaviors of the PMA. Normally, it should be sufficiently large, so that the proxy trajectory will track the reference accurately and realize a damped response.

This completes the proof.

**B. Adaptive Proxy-Based Robust Control Integrated With Nonlinear Disturbance Observer**

The previous analysis indicates that the APRC can suppress system uncertainties and ensure the uniformly ultimately boundedness of the system states. However, unlike fast-response motors, the response of the PMA system tends to be relatively slow. The excessive gain $\Gamma^*$ leads to control accuracy degradation and system instability. Therefore, a nonlinear disturbance observer is considered to handle the system uncertainties and increase the system robustness. According to Lemma 1 and Assumption 1, we have
\[
|\hat{\tau}| \leq \mu \tag{36}
\]
where $\mu$ is an unknown constant.
We define an auxiliary variable $z$ to design the nonlinear disturbance observer, as shown
\[
\begin{align*}
\hat{\tau} &= z + \kappa \dot{x} \\
\dot{z} &= -\kappa (f(x, \dot{x}) + b(x, \ddot{x})u + \hat{\tau}) \tag{37}
\end{align*}
\]
where $\hat{\tau}$ is the estimation of disturbances and $\kappa$ is a constant gain. Therefore, the derivative of $\hat{\tau}$ is
\[
\dot{\hat{\tau}} = \dot{z} + \kappa \ddot{x} = \kappa \dot{\tau} \tag{38}
\]
where $\dot{\tau} = \tau - \hat{\tau}$. Subtracting both sides of (38) from $\dot{\tau}$, we have
\[
\dot{\tau} = \dot{\tau} - \kappa \dot{\tau} \tag{39}
\]
Defining a Lyapunov function
\[
V_\tau(\dot{\tau}) = \frac{1}{2} \dot{\tau}^2 \tag{40}
\]
and evaluating $\dot{V}_\tau(\dot{\tau})$ along (39)
\[
\dot{V}_\tau(\dot{\tau}) = \dot{\tau} \dot{\tau} = \dot{\tau}(\dot{\tau} - \kappa \dot{\tau}) \leq \mu |\dot{\tau}| - \kappa |\dot{\tau}|^2 \leq -|\dot{\tau}| (|\dot{\tau}| - \mu). \tag{41}
\]
Therefore, the estimation error is bounded by
\[
|\dot{\tau}| \leq \hat{\varepsilon} \tag{42}
\]
where $\hat{\varepsilon} = \mu / \kappa$.

To integrate the nonlinear disturbance observer into the APRC, we only need to redefine (5) as
\[
\dot{S}_q + K_p (x_p - x) + K_i \int (x_p - x) \, dt + K_d (x_p - \dot{x}) + \hat{\tau} = 0. \tag{43}
\]
Similarly, bringing (2) and (3) into (42), the control signal of the PMA system is
\[
u = \frac{1}{b(x, \dot{x})} [\ddot{x}_d + c_1 (\dot{x}_d - \dot{x}) + c_2 (x_d - x) - f(x, \dot{x}) + K_p (x_p - x) \int (x_p - x) \, dt + K_d (x_p - \dot{x}) + \hat{\tau}]. \tag{44}
\]

**Theorem 2:** The norm of $\bar{e}_p = [e_p, \dot{e}_p, \ddot{e}_p, \tilde{\tau}]^T$ is uniformly ultimately bounded, and a sliding motion on the surface (4) can be guaranteed when the APRC–NDO satisfies
\[
m_p > 0, \lambda(K_c) > 0, \Gamma^* \geq \lambda_2(K_p + K_i + K_d), \omega > 0 \tag{45}
\]
where $K'_m = \text{diag} \{K_i, c_2, \omega, K_d, \kappa\}$, and
\[
\lambda_2 = \frac{\hat{\varepsilon} (c_1 + c_2) + \mu}{\lambda_{\min}(K'_m)}. \tag{46}
\]

**Proof:** We define a new Lyapunov candidate
\[
V' = V_1 + V_2 + V_3 + \frac{1}{2} \hat{\tau}^2 \tag{47}
\]
and note that $\Gamma^* \geq \lambda_2(K_p + K_i + K_d) \geq \hat{\varepsilon}$. From (16), (21), (36)–(39), and (42), the derivative of $V'$ is
\[
\dot{V}' = -\Gamma^* |S_p| - \tilde{\tau} S_q - K_d \ddot{e}_p^2 - \kappa \dot{e}_p^2 - K_i (c_2 e_p^2 + \ddot{\tau}) \leq -\hat{\varepsilon} |S_q| + \hat{\varepsilon} |\dot{e}_p| + c_1 |\dot{e}_p| + c_2 |e_p| - \tilde{\tau} S_q - K_d \ddot{e}_p^2 - \kappa \dot{e}_p^2 - K_i (c_2 e_p^2 + \mu |\tilde{\tau}|) - \kappa |\dot{\tau}|^2 \leq [\bar{e}_p |(c_1 + c_2) + \mu|] ||\bar{e}_p|| - \lambda_{\min}(K'_m) ||\bar{e}_p||^2 \leq -||\bar{e}_p|| (\lambda_{\min}(K'_m) ||\bar{e}_p|| - ||\bar{e}_p|| |[\bar{e}_p]| + \mu (c_1 + c_2) + \mu)). \tag{48}
\]
Thus, $\bar{e}_p$ is uniformly ultimately bounded by
\[
||\bar{e}_p|| \leq \lambda_2. \tag{49}
\]
frequency sinusoid

displacement sensor was GA-75 whose measurement range was
length of 200 mm, and an operating pressure range from 0 to
compressed air and was connected to the PMA through the elec-
trol signal to an electromagnetic proportional valve for regulat-
ing the inner pressure of the PMA. The air compressor provided
A/D and D/A to collect the sensory data and transmitted the con-

Fig. 3. PMA system.

In this situation, by applying the similar technique in (27), the
achievement of a sliding motion on (4) is guaranteed.

\[ V_p' = \frac{1}{2} m_p \dot{s}_p^2 + \frac{1}{2} \dot{\gamma}^2. \]  (47)

Thus, the derivative of \( V_p' \) is expressed as

\[ \dot{V}_p = m_p \dot{s}_p \ddot{s}_p + \frac{1}{2} \dot{\gamma}^2 \]
\[ \leq -\Gamma' |s_p| + \lambda^2(K_p + K_i + K_d)s_p \]
\[ \leq 0. \]  (48)

This completes the proof.

**Corollary 2:** If inequality (46) holds, and initially \( x_p = x_d \),
then, as the proxy mass \( m_p \) increases, \( S_q \) will gradually ap-
proach a bound associated with estimation errors of the lumped
disturbances.

\[ \lim_{m_p \to \infty} |s_q| \leq \lambda^2(c_1 + c_2 + 1). \]  (49)

**Proof:** This corollary can be easily proven by using the
similar method given in the Proof of Corollary 1.

**IV. EXPERIMENTS**

**A. Experiment Setup**

In the physical system, the board (NI-PCI 6052E) enabled
A/D and D/A to collect the sensory data and transmitted the control
signal to an electromagnetic proportional valve for regulating
the inner pressure of the PMA. The air compressor provided
compressed air and was connected to the PMA through the elec-
tromagnetic proportional valve. Consequently, the displacement
of the PMA was Festo DMSP-20-200N-RM-RM fluidic muscle with an internal diameter of 20 mm, nominal
length of 200 mm, and an operating pressure range from 0 to
6 bar. The Festo VPPM-6L-L-1G18-0L10H-V1P proportional
valve was used to regulate the pressure inside the PMA. The
displacement sensor was GA-75 whose measurement range was
0–150 mm.

The proposed method does not require an accurate three-
element model of the PMA. So, we used the identified parameters of a similar PMA in [11] (see Table I).

We designed two reference trajectories. The first was a fixed
frequency sinusoid

\[ x_d = A_x \sin(2\pi f_x t) + B_x \]  (50)

where \( A_x = 0.015 \) m, \( f_x = 0.25 \) Hz, and \( B_x = 0.015 \) m. The second was a sine wave whose frequency changed linearly from
0.1 to 0.5 Hz within 20 s. The sampling time was set to 0.001 s.

The maximum absolute error (MAE), the integral of absolute
error (IAE), and the relative tracking accuracy (RTE) were used as our performance measurements

\[ MAER^a = \max (|x_d(t) - x(t)|) \]  (51)
\[ IAE^b = \frac{1}{N} \sum_{t=1}^{N} |x_d(t) - x(t)| \]  (52)
\[ RTE^c = \left( \frac{\sum_{t=1}^{N} |x_d(t) - x(t)|}{N} \right) \times 100\% \]  (53)

where \( N \) is the total sampling time. \( x_a \) is the maximum running
displacement of the PMA.

The following set of control parameters of the APRC–NDO
was used in all experiments: \( c_1 = 177.4, c_2 = 174.4, K_p =
2473.5, K_i = 1916, K_d = 194.2, \kappa = 15952, \gamma = 10 \). Note
that the parameter selections of all the control strategies were
based on an optimization algorithm, called switch-mode firefly
algorithm (SMFA). More related details can be found in [30].

**B. Experimental Results**

Fig. 4 shows the experimental results that verified Corollary 1.
In this experiment, we selected a fixed \( \Gamma^* \) to demonstrate the influence of \( m_p = \{0.5, 1.0, 5.0, 10.0, 15.0\} \). As \( m_p \) increased,
the tracking accuracy improved, and the variation of \( S_q \) signifi-
cantly decreased. Meanwhile, \( x_p \) tracked the reference trajectory
more accurately and \( |s_q| \to 0 \).

Then, we intended to verify the experimental results of the
proposed control strategy with different amplitudes of the de-
sired trajectories, as shown in Fig. 5. The corresponding control
performances were similar, and the control parameters did not change for this experiment, which indicated that the proposed method is applicable to various applications.

Next, for a fair comparison, the control parameters for all the
strategies [APRC–NDO, NDO–SMC, super twisting algorithm
(STA), PSMC] were adjusted with the fixed-frequency sinu-
soidal reference (\( f_x = 0.25 \) Hz, \( A_x = 0.015 \) m, \( B_x = 0.015 \) m).
by the SMFA. Fig. 6 shows the corresponding performance of
different control strategies, and the corresponding MAEs, IAEs,
and RTEs of all five control strategies are shown in Table II.
We replaced the sign function of the NDO–SMC with a sat
function to eliminate chattering. In spite of the inaccurate model
parameters in Table I, the NDO–SMC and STA were capable of
handling the uncertainties and achieving favorable performance.
Meanwhile, the basic PSMC enabled the PMA to track the

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (Unit)</th>
<th>Parameter</th>
<th>Value (Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0 )</td>
<td>-202.32 (N)</td>
<td>( f_1 )</td>
<td>0.00721 (N/Pa)</td>
</tr>
<tr>
<td>( k_{01} )</td>
<td>18063.0 (N/m)</td>
<td>( k_{02} )</td>
<td>0.01051 (N/m/Pa)</td>
</tr>
<tr>
<td>( k_{11} )</td>
<td>-0.2132 (N/m)</td>
<td>( k_{12} )</td>
<td>9063.80 (N/m/Pa)</td>
</tr>
<tr>
<td>( b_{01} )</td>
<td>6435.31 (N/s/m)</td>
<td>( b_{11} )</td>
<td>0.10023 (N/s/m/Pa)</td>
</tr>
<tr>
<td>( b_{0d} )</td>
<td>2522.01 (N/s/m)</td>
<td>( b_{1d} )</td>
<td>0.00321 (N/s/m/Pa)</td>
</tr>
</tbody>
</table>

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reference with acceptable precision, since it is a model-free strategy, not affected by inaccurate model parameters. However, its performance was still unsatisfied than APRC–NDO. Besides, according to our previous study [8], although the traditional PID controller tracked the reference trajectory, the performance was worse than the proposed strategy. On the other hand, because we had $|s_p| \rightarrow 0$, the adaptive coefficient $\hat{\Gamma}$ gradually tended to be a fixed value, instead of monotonically increasing.

Based on the previous tuned control parameters, Fig. 7 showed the tracking performance of different control strategies with the varying-frequency (0.1–0.5 Hz) sinusoidal reference. Although the NDO–SMC behaved well in the steady state, it had a large oscillation at the beginning. On the contrary, the STA performed well at the beginning, but it could not effectively track the reference as the frequency increased. Still, the PSMC tracked the reference with acceptable precision. The proposed APRC–NDO performed the best among all four control strategies, although it had a relatively little oscillation at the beginning. This was because the APRC–NDO needed some time to drive the states of the PMA into the boundary [see (25) and (46)]. After that, the proposed APRC–NDO could handle the system disturbances/uncertainties and achieve accurate tracking. Actually, the NDO is very important, because the uncertainties/disturbances of the system relate to the frequency of PMA’s trajectory, and
the varying frequency causes the growth of the system’s uncertainties/disturbances. The SMC and APRC integrated with NDO can better handle the uncertainties/disturbances of the system. To further investigate the robustness of the proposed control strategy, we first designed an experiment, in which a sudden change of the load (2.5-kg load or 5.0-kg load) was added to the PMA during operation, as shown in Fig. 8. It is seen that the trajectory of the PMA deviated significantly from the reference, and the greater the sudden disturbance, the further it deviated. Moreover, additional experiments were conducted for tracking the varying-frequency (0.1–0.5 Hz) reference with different loads, as shown in Fig. 9. Generally, they were very robust to the changing loads. However, the fixed parameters of the NDO can only handle a certain amount of disturbances. When the disturbance is beyond a certain degree, the parameters of NDO have to be retuned.

V. CONCLUSION

This article presented a robust control strategy, APRC–NDO, for the PMA. The APRC–NDO can realize a damped response and regulate the behaviors of the PMA via a virtual proxy, as well as handle the system uncertainties/disturbances to improve the robustness. The tracking states of the PMA were proven to be uniformly ultimately bounded through two motion phases. Finally, extensive experiments demonstrated the superior performance of the APRC–NDO.

REFERENCES

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