Social Judgment Advisor: An Application of the Perceptual Computer

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***Abstract*— The Perceptual Computer (Per-C) is an architecture for making subjective judgments by computing with words. An application of the Per-C to a social judgment is described in this paper. First, a vocabulary is established for the social judgment and its words are modeled by interval type-2 fuzzy sets (IT2 FSs). Surveys are then designed to establish a structure of the rulebase and to obtain a rule-consequent histograms. After pre-processing to remove bad responses and outliers, perceptual reasoning (PR) is used to simplify the rulebase. Once the rulebase is established, PR is also used to infer the output IT2 FSs for new inputs. Finally, the output IT2 FSs are mapped back into words in the codebook using a similarity measure. So, from a user's point of view, he or she is interacting with the Per-C using only words from a vocabulary. The techniques introduced in this paper should be applicable to many rule-based decision-making situations.**

I. Introduction

Zadeh coined the phrase "*computing with words*" (CWW) [28], [29]. According to [29], CWW is *"a methodology in which the objects of computation are words and propositions drawn from a natural language."* There are at least two types of uncertainties associated with a word [18]: intra-personal uncertainty and inter-personal uncertainty. The former is explicitly pointed out by Wallsten and Budescu [18] as *"except in very special cases, all representations are vague to some degree in the minds of the originators and in the minds of the receivers,"* and they suggest to model it by type-1 fuzzy sets (T1 FSs). The latter is pointed out by Mendel [8] as *"words mean different things to different people"* and Wallsten and Budescu [18] as *"different individuals use diverse expressions to describe identical situations and understand the same phrases differently when hearing or reading them."* Because an interval type-2 FS (IT2 FS) can be viewed as a group of T1 FSs, it can model both types of uncertainty; hence, we suggest IT2 FSs be used in CWW [7], [8], [10]. An example of a trapezoidal IT2 FS is shown in Fig. 1. It is characterized by a footprint of uncertainty (FOU), which is bounded by an upper membership function (UMF), $\bar{A}$, and a lower membership function (LMF), $\underline{A}$.

A specific architecture is proposed in [9] for making subjective judgments by CWW, and is shown in Fig. 2. It is called a *Perceptual Computer*—Per-C for short. In Fig. 2, the *encoder*[1] transforms linguistic perceptions into IT2 FSs

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[1]Zadeh calls this *constraint explicitation* in [28], [29]. In [30], [31] and some of his recent talks, he calls this *precisiation*.

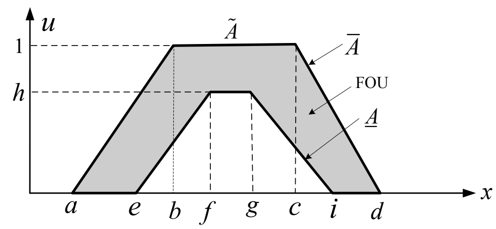

Fig. 1. A trapezoidal IT2 FS. (a, b, c, d) determines a normal trapezoidal UMF, and (e, f, g, i, h) determines a trapezoidal LMF with height h.

that activate a *CWW engine*. The CWW engine performs operations on the IT2 FSs. The *decoder*[2] maps the output of the CWW engine into a recommendation, which can be a word, rank, or class.

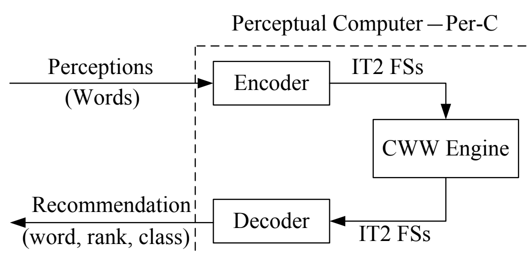

Fig. 2. Conceptual structure of the Perceptual Computer.

To operate the Per-C, one needs to solve the following problems:

1) *How to transform words into IT2 FSs, i.e., the encoding problem.* This can be done with Liu and Mendel's *Interval Approach* [5]. First, for each word in an application-dependent encoding vocabulary, a group of subjects are asked the following question:

   On a scale of 0-10, what are the end-points of an interval that you associate with the word ____?

   After some pre-processing, during which some intervals (e.g., outliers) are eliminated, each of the remaining intervals is classified as either an interior, left-shoulder or right-shoulder IT2 FS. Then, each of the word's data intervals is individually mapped into its respective T1 interior, left-shoulder or right-shoulder MF, after which the union of all of these T1 MFs is taken. The result is an FOU for an IT2 FS model of the word. The words and their FOUs constitute a *codebook*.

[2]Zadeh calls this *linguistic approximation* in [28], [29].

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2) *How to construct the CWW engine, which maps IT2 FSs into IT2 FSs.* There are different kinds of CWW engines, e.g.,

   a) The *linguistic weighted average* [21], [23], which is defined as

$$\tilde{Y} = \frac{\sum_{i=1}^{N} \tilde{X}_i \tilde{W}_i}{\sum_{i=1}^{N} \tilde{W}_i} \quad (1)$$

   where $\tilde{X}_i$, the sub-criteria (e.g., data, features, decisions, recommendations, judgments, scores, etc), and $\tilde{W}_i$, the weights, are usually words modeled by IT2 FSs; however, they can also be special cases of IT2 FSs, e.g., numbers, intervals, or T1 FSs. It has been shown [21], [23] that the UMF of $\tilde{Y}$ is a fuzzy weighted average [4] of the UMFs of $\tilde{X}_i$ and $\tilde{W}_i$, and the LMF of $\tilde{Y}$ is a fuzzy weighted average of the LMFs of $\tilde{X}_i$ and $\tilde{W}_i$.

   b) *Perceptual reasoning (PR)* [26], which considers the following problem:
   Given a rulebase with N rules, each of the form:

$$R^i : \textit{If } x_1 \textit{ is } \tilde{F}_1^i \textit{ and } \ldots \textit{ and } x_p \textit{ is } \tilde{F}_p^i, \quad \textit{Then } y \textit{ is } \tilde{G}^i \quad (2)$$

   where $\tilde{F}_j^i$ and $\tilde{G}^i$ are words modeled by IT2 FSs, and a new input $\tilde{\mathbf{X}}' = (\tilde{X}_1, \ldots, \tilde{X}_p)$, where $\tilde{X}_j$ $(j = 1, \ldots, p)$ are also words modeled by IT2 FSs, then what is the output IT2 FS $\tilde{Y}_{PR}$?
   In PR [26] one computes

$$\tilde{Y}_{PR} = \frac{\sum_{i=1}^{N} f^i \tilde{G}^i}{\sum_{i=1}^{N} f^i} \quad (3)$$

   where f^i is the firing level of R^i, i.e.,

$$f^i = \prod_{j=1}^{p} s_J(\tilde{X}_j, \tilde{F}_j^i) \quad (4)$$

   in which $s_J(\tilde{X}_j, \tilde{F}_j^i)$ is the Jaccard similarity for IT2 FSs [24] given in (5) on top of the next page. Another approach that uses firing intervals instead of firing levels is described in [12]. By using PR $\tilde{Y}_{PR}$ is guaranteed to look like the codebook FOUs [12], [26].

3) *How to map the output of the CWW engine into a recommendation, i.e., the decoding problem.* Thus far, there are three kinds of decoders according to three forms of recommendations:

   a) *Word:* To map an IT2 FS into a word, it must be possible to compare the *similarity* between two IT2 FSs. The Jaccard similarity measure [24] described by (5) can be used to compute the similarities between the CWW engine output and all words in the codebook. Then, the word with the maximum similarity is chosen as the Decoder's output.

   b) *Rank:* Ranking is needed when several alternatives are compared to find the best. Because the performance of each alternative is represented by an IT2 FS obtained from the CWW engine, a ranking method for IT2 FSs is needed. A centroid-based ranking method for IT2 FSs is described in [24].

   c) *Class:* A classifier is necessary when the output of the CWW engine needs to be mapped into a decision category [13]. Subsethood [14], [17], [20], [27] is useful for this purpose. One first computes the subsethood of the CWW engine output for each of the possible classes. Then, the final decision class is the one corresponding to the maximum subsethood.

In this paper the Per-C is designed as an aid for making social judgments. The result is called a *Social Judgment Advisor* (SJA). By "judgment" is meant an assessment of the *level* of a variable of interest. Particularly, an SJA is developed for flirtation judgments [6] based on IF-THEN rules that are obtained from people, the result being a *Fuzzy Logic Flirtation Advisor* (FLFA). Flirtation judgments offer a fertile starting place for developing an SJA for a variety of reasons. First, many behavioral indicators associated with flirtation have been well established [3]. Second, the indicators (e.g., smiling, touching, eye contact) are often ambiguous by themselves and along with a changing level of the behavior (along with other cues) the meaning of the behavior is apt to shift from one inference (e.g., friendly) to another (e.g., flirtation, seductive, or harassing). Third, participants are apt to have had a great deal of experience with flirtation judgments, and be therefore apt to easily make them. Finally, inferences made about the meaning of these behaviors are often sensitive to both the gender of the perceiver and the gender of the interactants [3].

The rest of this paper is organized as follows: Section II describes the methodology to design an SJA. Section III uses the FLFA as an example to illustrate how an SJA can be designed and used. Finally, Section IV draws conclusions.

II. Design An SJA

In this section the complete procedure for designing an SJA is described. Although the focus is on flirtation judgment, the methodology can also be applied to engineering judgments such as global warming, environmental impact, water quality, audio quality, toxicity, etc.

A. Survey Design

An SJA uses a rulebase, which is obtained from surveys. The following methodology can be used to conduct the surveys [8], [11]:

1) *Identify the behavior of interest.* This step, although obvious, is highly application dependent. As mentioned above, our focus is on the behavior of flirtation.
2) *Determine the indicators of the behavior of interest.* This requires:

$$s_J(\tilde{X}_j, \tilde{F}^i_j) = \frac{\int_X \min(\overline{X}_j(x), \overline{F}^i_j(x))dx + \int_X \min(\underline{X}_j(x), \underline{F}^i_j(x))dx}{\int_X \max(\overline{X}_j(x), \overline{F}^i_j(x))dx + \int_X \max(\underline{X}_j(x), \underline{F}^i_j(x))dx}. \tag{5}$$

a) Establishing a list of candidate indicators (e.g., for flirtation [11], six candidate indicators are *touching*, *eye contact*, *acting witty*, *primping*, *smiling*, and *complementing*).

b) Conducting a survey in which a representative population is asked to rank-order in importance the indicators on the list of candidate indicators. In some applications it may already be known what the relative importance of the indicators is, in which case a survey is not necessary.

c) Choosing a meaningful subset of the indicators, because not all of them may be important. In Step 6, where people are asked to provide consequents for a collection of IF–THEN rules by means of a survey, the survey must be kept manageable, because most people do not like to answer lots of questions; hence, it is very important to focus on the truly significant indicators. The analytic hierarchy process [16] and factor analysis [1] from statistics can be used to help establish the relative significance of indicators.

3) *Establish scales for each indicator and the behavior of interest.* If an indicator is a physically measurable quantity (e.g., temperature, pressure), then the scale is associated with the expected range between the minimum and maximum values for that quantity. On the other hand, many social judgment indicators as well as the behavior of interest are not measurable by means of instrumentation (e.g., touching, eye contact, flirtation, etc.). Such indicators and behaviors need to have a scale associated with them, or else it will not be possible to design or activate an SJA. Commonly used scales are 1 through 5, 0 through 5, 0 through 10, etc. We shall use the scale 0 through 10.

4) *Establish names and collect interval data for each of the indicator's FSs and behavior of interest's FSs*. The issues here are:

a) What vocabulary should be used and what should its size be so that the FOUs for the vocabulary completely cover the 0-10 scale and provide the user of the SJA with a user-friendly interface?

b) What is the smallest number of FSs that should be used for each indicator and behavior of interest for establishing rules?

This is the encoding problem and the IA [5] can be used to find the FOU word models once a satisfactory vocabulary has been established, and word data have been collected from a group of subjects using surveys.

5) *Establish the rules.* Rules are the heart of the SJA; they link the indicators of a behavior of interest to that behavior. The following issues need to be addressed:

a) How many antecedents will the rules have? As mentioned earlier, people generally do not like to answer complicated questions; so, we advocate using rules that have either one or two antecedents. An interesting (non-engineering) interpretation for a two-antecedent rule is that it provides the *correlation* effect that exists in the mind of the survey respondent between the two antecedents. Psychologists have told us that it is just about impossible for humans to correlate more than two antecedents (indicators) at a time, and that even correlating two antecedents at a time is difficult. Using only one or two antecedents does not mean that a person does not use more than this number of indicators to make a judgment; it means that a person uses the indicators one or two at a time (this should be viewed as a *conjecture*). This suggests the overall architecture for the SJA should be parallel or hierarchical.

b) How many rulebases need to be established? Each rulebase has its own SJA. When there is more than one rulebase, each of the advisors is a social judgment sub-advisor, and the outputs of these sub-advisors can be combined to create the structure of the overall SJA. If, e.g., it has been established that four indicators are equally important for the judgment of flirtation, then there would be up to four single-antecedent rulebases as well as six two-antecedent rulebases. These rulebases can be rank-ordered in importance by means of another survey in which the respondents are asked to do this. Later, when the outputs of the different rulebases are combined, they can be weighted using the results of this step.

There is a very important reason for using sub-advisors for an SJA. Even though the number of important indicators has been established for the social judgment, it is very unlikely that they will all occur at the same time in a social judgment situation. If, for example, *touching, eye contact, acting witty* and *primping* have been established as the four most important indicators for flirtation, it is very unlikely that in a new flirtation scenario, all four occur simultaneously. From your own experiences in flirting, can you recall a situation when someone was simultaneously touching you, made eye contact with you, was acting witty and was also primping? Not very likely! Note that a missing observation is not the same as an observation of zero value; hence, even if it was possible to create four antecedent rules, none of those rules could be activated if one or more of the indicators had a missing observation. It is therefore very important to have sub-advisors that will be activated when one or two of these indicators are occurring.

More discussions about this can be found in [14], [20].

6) *Survey people (experts) to provide consequents for the rules*. If, e.g., a single antecedent has five FSs associated with it, then respondents would be asked five questions. For two-antecedent rules, where each antecedent is again described by five FSs, there would be 25 questions. The order of the questions should be randomized so that respondents don't correlate their answers from one question to the next. In Step 4 earlier, the names of the consequent FSs were established. Each single-antecedent rule is associated with a question of the form:

IF the antecedent is (one of the antecedent's FSs), THEN there is (one of the consequent's FSs) of flirtation.

Each two-antecedent rule is associated with a question of the form:

IF antecedent 1 is (one of antecedent 1's FSs) and antecedent 2 is (one of antecedent 2's FSs), THEN there is (one of the consequent's FSs) of flirtation.

The respondent is asked to choose one of the given names for the consequent's FSs. The rulebase surveys will lead to rule consequent histograms, because everyone will not answer a question the same way.

The following nine terms, shown in Fig. 3, are taken from the 32-word vocabulary[3] in [5], and are used as the codebook for the SJA: *none to very little* (NVL), *a bit* (AB), *somewhat small* (SS), *some* (S), *moderate amount* (MOA), *good amount* (GA), *considerable amount* (CA), *large amount* (LA), and *maximum amount* (MAA). Table I summarizes the FOUs and centroids[4] of these words. These FOUs are being used only to illustrate our SJA methodology. In actual practice, word survey data would have to be collected from a group of subjects, using the words in the context of flirtation.

Because of the page limit, in this paper our SJA is limited to rulebases for one-antecedent rules, in which x denotes touching, and y denotes flirtation level. Two-antecedent SJAs, as well as how to deduce the output for multiple antecedents using rulebases consisting of only one or two antecedents, can be found in [14], [20]. For all of the rules, the following five-word subset of the codebook was used for both their antecedents and consequents: *none to very little*, *some*, *moderate amount*, *large amount*, and *maximum amount*. It is easy to see from Fig. 3 that these words cover the interval [0, 10]. Table II, which is taken from [11] and Chapter 4 of [8], provides the data collected from 47 respondents to the Step 6 surveys.

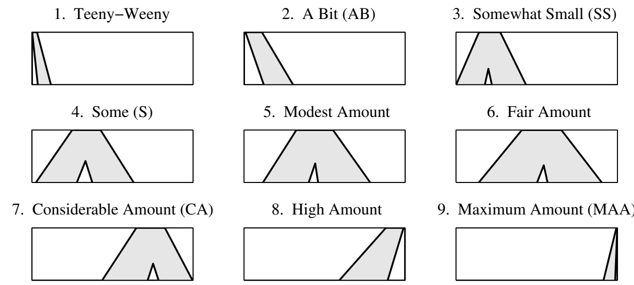

Fig. 3. Nine word FOUs ranked by their centers of centroid. Words 1, 4, 5, 8 and 9 were used in the Step 6 survey.

TABLE II

HISTOGRAM OF SURVEY RESPONSES FOR SINGLE-ANTECEDENT RULES BETWEEN TOUCHING LEVEL AND FLIRTATION LEVEL. ENTRIES DENOTE THE NUMBER OF RESPONDENTS OUT OF 47 THAT CHOSE THE CONSEQUENT.

Touching	Flirtation				
	NVL	S	MOA	LA	MAA
1. NVL	42	3	2	0	0
2. S	33	12	0	2	0
3. MOA	12	16	15	3	1
4. LA	3	6	11	25	2
5. MAA	3	6	8	22	8

B. Data Pre-Processing

Inevitably, there are bad responses and outliers in the survey histograms. These bad data need to be removed before the histograms are used.

Data pre-processing consists of three steps: 1) bad data processing, 2) outlier processing, and, 3) tolerance limit processing, which are quite similar to the pre-processing steps used in [5]. Rule 1 in Table II is used below as an example to illustrate the details of these three steps. The number of responses before pre-processing are shown in the first row of Table III.

1) Bad Data Processing: This removes gaps (a zero between two non-zero values) in a group of subject's responses. For Rule 1 in Table II, the number of responses to the five consequents are $\{42, 3, 2, 0, 0\}$. Because there is no gap among these numbers, no response is removed, as shown in the second row of Table III. On the other hand, for Rule 2 in Table II, the numbers of responses to the five consequents are $\{33, 12, 0, 2, 0\}$. Observe that no respondent selected the word *MOA* between *S* and *LA*; hence, a gap exists between

[3] They are selected in such a way that they are distributed somewhat uniformly in $[0, 10]$.

[4] The centroid [2], [8] of an IT2 FS $\tilde{A}$ is an interval $C_{\tilde{A}} = [c_l, c_r]$, where

$$c_l = \min_{\mu(x)\in[\mu_{\underline{A}}(x),\mu_{\bar{A}}(x)]} \frac{\int_X x\mu(x)dx}{\int_X \mu(x)dx}$$

$$c_r = \max_{\mu(x)\in[\mu_{\underline{A}}(x),\mu_{\bar{A}}(x)]} \frac{\int_X x\mu(x)dx}{\int_X \mu(x)dx}$$

and the center of centroid [24] is $c_{\tilde{A}} = (c_l+c_r)/2$. KM or EKM Algorithms [8], [22], [25] are used to compute c_l and c_r.

TABLE I

FOU DATA FOR THE 9-WORD CODEBOOK. AS SHOWN IN FIG. 1, EACH UMF IS REPRESENTED BY (a, b, c, d), AND EACH LMF IS REPRESENTED (e, f, g, i, h).

Word	**UMF**	**LMF**	**Centroid** [2]	**Center of Centroid** [24]
1. None to Very Little (NVL)	[0, 0, 0.14, 1.97]	[0, 0, 0.05, 0.66, 1]	[0.22, 0.73]	0.48
2. A Bit (AB)	[0.58, 1.50, 2.00, 3.41]	[0.79, 1.68, 1.68, 2.21, 0.74]	[1.42, 2.09]	1.76
3. Somewhat Small (SS)	[0.59, 2.00, 3.25, 4.41]	[2.29, 2.70, 2.70, 3.21, 0.42]	[1.75, 3.43]	2.59
4. Some (S)	[0.38, 2.50, 5.00, 7.83]	[2.88, 3.61, 3.61, 4.21, 0.35]	[2.03, 5.78]	3.91
5. Moderate Amount (MOA)	[2.59, 4.00, 5.50, 7.62]	[4.29, 4.75, 4.75, 5.21, 0.38]	[3.73, 6.16]	4.95
6. Good Amount (GA)	[3.38, 5.50, 7.50, 9.62]	[5.79, 6.50, 6.50, 7.21, 0.41]	[5.11, 7.89]	6.50
7. Considerable Amount (CA)	[4.38, 6.50, 8.25, 9.62]	[7.19, 7.58, 7.58, 8.21, 0.37]	[5.97, 8.52]	7.25
8. Large Amount (LA)	[5.98, 7.75, 8.60, 9.52]	[8.03, 8.37, 8.37, 9.17, 0.57]	[7.50, 8.75]	8.13
9. Maximum Amount (MAA)	[8.68, 9.91, 10, 10]	[9.61, 9.97, 10, 10, 1]	[9.51, 9.87]	9.69

S and *LA*. Let $G_1 = \{NVL,\ S\}$ and $G_2 = \{LA\}$. Because G_1 has more responses than G_2, it is passed to the next step of data pre-processing and G_2 is discarded.

2) Outlier processing: Outlier processing uses a Box and Whisker test [19]. As explained in [5], outliers are points that are unusually too large or too small. A Box and Whisker test is usually stated in terms of first and third quartiles and an interquartile range. The first and third quartiles, $Q(0.25)$ and $Q(0.75)$, contain 25% and 75% of the data, respectively. The inter-quartile range, IQR, is the difference between the third and first quartiles; hence, IQR contains 50% of the data between the first and third quartiles. Any datum that is more than $1.5\,IQR$ above the third quartile or more than $1.5\,IQR$ below the first quartile is considered an outlier [19].

Rule consequents are words modeled by IT2 FSs; hence, the Box and Whisker test cannot be directly applied to them. In our approach, the Box and Whisker test is applied to the *set of centers of centroids* (see Footnote 4) formed by the centers of centroids of the rule consequents. Focusing again on Rule 1 in Table II, the centers of centroids of the consequent IT2 FSs *NVL, S, MOA, LA* and *MAA* are first computed (see the last column of Table I), and are 0.48, 3.91, 4.95, 8.13 and 9.69, respectively. Then the set of centers of centroids is

$$\{\underbrace{0.48, \cdots, 0.48}_{42}, \underbrace{3.91, 3.91, 3.91}_{3}, \underbrace{4.95, 4.95}_{2}\} \tag{6}$$

where each center of centroid is repeated a certain number of times according to the number of respondents after bad data processing. The Box and Whisker test is then applied to this crisp set, where $Q(0.25) = 0.48$, $Q(0.75) = 0.48$, and $1.5\,IQR = 0$. For Rule 1, the three responses to *S* and the two responses to *MOA* are removed, as shown in the third row of Table III. The new set of centers of centroids becomes

$$\{\underbrace{0.48, \cdots, 0.48}_{42}\} \tag{7}$$

3) Tolerance limit processing: Let m and σ be the mean and standard deviation of the remaining histogram data after outlier processing. If a datum lies in the tolerance interval $[m - k\sigma, m + k\sigma]$, then it is accepted; otherwise, it is rejected [19]. k is determined such that one is 95% confident that the given limits contain at least 95% of the available data, and it can be obtained from a table look-up [15].

For Rule 1 in Table II, tolerance limit processing is performed on the set of 42 centers of centroids in (7), for which $m = 0.48$, $\sigma = 0$ and $k = 2.43$. No word is removed for this particular example; so, only one consequent, *NVL*, is accepted for this rule, as shown in the last row of Table III.

The final pre-processed responses for the histograms in Table II are given in Table IV. Observe that most responses have been preserved.

TABLE III

DATA PRE-PROCESSING RESULTS FOR THE 47 RESPONSES TO THE QUESTION *"IF there is* NVL *touching, THEN there is _______ flirtation."*

Number of responses	NVL	S	MOA	LA	MAA
Before pre-processing	42	3	2	0	0
After bad data processing	42	3	2	0	0
After outlier processing	42	0	0	0	0
After tolerance limit processing	42	0	0	0	0

TABLE IV

PRE-PROCESSED HISTOGRAMS OF TABLE II.

Touching	**Flirtation**				
	NVL	**S**	**MOA**	**LA**	**MAA**
1. NVL	42	0	0	0	0
2. S	33	12	0	0	0
3. MOA	12	16	15	3	0
4. LA	0	6	11	25	2
5. MAA	0	6	8	22	8

C. Rulebase Generation

Observe from Table IV that the survey and data pre-processing lead to rule consequent histograms, but how the histograms should be used is an open question. In [8] three possibilities were proposed:

1) Keep the response chosen by the largest number of respondents.
2) Find a weighted average of the rule consequents for each rule.

3) Preserve the distributions of the expert-responses for each rule.

Clearly, the disadvantage of keeping the response chosen by the largest number of respondents is that this ignores all the other responses.

The second method was studied in detail in [8]. Using that method, when T1 FSs were used (see Chapter 5 of [8]), the consequent for each rule was a crisp number, c, where

$$c = \frac{\sum_{m=1}^{5} c_m w_m}{\sum_{m=1}^{5} w_m} \tag{8}$$

in which c_m is the centroid [8] of the m^{th} T1 consequent FS, and w_m is the number of respondents for the m^{th} consequent. When IT2 FSs were used (see Chapter 10 of [8]), the consequent for each rule was an interval, C, where

$$C = \frac{\sum_{m=1}^{5} C_m w_m}{\sum_{m=1}^{5} w_m} \tag{9}$$

in which C_m is the centroid [2], [8] of the m^{th} IT2 consequent FS.

The disadvantages of using (8) or (9) are: (1) there is information lost when converting the T1 or IT2 consequent FSs into their centroids, and (2) it is difficult to describe the aggregated rule consequents (c or C) linguistically.

Our approach is to preserve the distributions of the expert-responses for each rule by using a different weighted average to obtain the rule consequents, as illustrated by the following:

Example 1: Observe from Table IV that when the antecedent is *some* (S) there are two valid consequents, so that the following two rules will be fired:

R_1^2: IF touching is *some*, THEN flirtation is *none to very little*.

R_2^2: IF touching is *some*, THEN flirtation is *some*.

These two rules should not be considered of equal importance because they have been selected by different numbers of respondents. An intuitive way to handle this is to assign weights to the two rules, where the weights are proportional to the number of responses, e.g., the weight for R_1^2 is $33/45 = 0.73$, and the weight for R_2^2 is $12/45 = 0.27$. The aggregated consequent $\tilde{Y}^2$ for R_1^2 and R_2^2 is

$$\tilde{Y}^2 = \frac{33NVL + 12S}{33 + 12} \tag{10}$$

The result is shown in Fig. 4. □

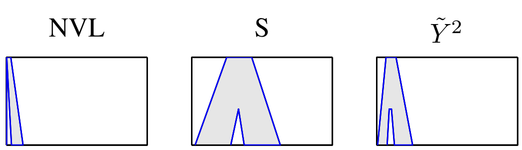

Fig. 4. $\tilde{Y}^2$ obtained by aggregating the consequents of R_1^2 (NVL) and R_2^2 (S).

Without loss of generality, assume there are N different combinations of antecedents (e.g., $N = 5$ for the single-antecedent rules in Table IV), and each combination has M possible different consequents (e.g., $M = 5$ for the rules in Table IV); hence, there can be as many as MN rules. Denote the m^{th} consequent of the i^{th} combination of the antecedents as $\tilde{Y}_m^i$ ($m = 1, 2, \ldots, M, i = 1, 2, \ldots, N$), and the number of responses to $\tilde{Y}_m^i$ as w_m^i. For each i, all M $\tilde{Y}_m^i$ can be combined first into a single IT2 FS using the following special case of the linguistic weighted average [14], [20], [21], [23]:

$$\tilde{Y}^i = \frac{\sum_{m=1}^{M} w_m^i \tilde{Y}_m^i}{\sum_{m=1}^{M} w_m^i} \tag{11}$$

$\tilde{Y}^i$ then acts as the (new) consequent for the i^{th} rule. By doing this, the distribution of the expert responses has been preserved for each rule. Examples of $\tilde{Y}^i$ for single-antecedent rules are depicted in Fig. 6(a), and are described in detail in Section III.

D. Computing the Output of the SJA

The previous subsection described how a simplified rule-base can be generated from a survey. In this subsection, we explain how PR is used to compute an output of the SJA for a new input $\tilde{\mathbf{X}}$. Because of the page limit, only single-antecedent rules are considered in this paper. For multi-antecedent rules, please refer to [14], [20].

Consider single-antecedent rules of the form

$$R^i: \text{ If } x \text{ is } \tilde{F}^i, \text{ Then } y \text{ is } \tilde{Y}^i \qquad i = 1, \ldots, N \tag{12}$$

where $\tilde{Y}^i$ are computed by (11). In PR, the Jaccard similarity measure (5) is used to compute the firing levels of the rules, i.e., $f^i = s_J(\tilde{X}, \tilde{F}^i)$, $i = 1, \ldots, N$. Once f^i are computed, the output FOU of the SJA is computed as [see (3)]

$$\tilde{Y}_C = \frac{\sum_{i=1}^{N} f^i \tilde{Y}^i}{\sum_{i=1}^{N} f^i} \tag{13}$$

The subscript *C* in $\tilde{Y}_C$ stands for *consensus* because $\tilde{Y}_C$ is obtained by aggregating the survey results from a population of people, and the resulting SJA is called a *consensus SJA*. Because only the nine words in Fig. 3 are used in the SJAs, the similarities among them can be pre-computed. Finally, $\tilde{Y}_C$ is mapped into a word in the Fig. 3 vocabulary also using the Jaccard similarity measure.

III. Examples of How to Use An SJA

As mentioned below (13), each SJA that is designed from survey is referred to as a *consensus SJA*, because it is obtained by using survey results from a group of people. Fig. 5 depicts[5] one way to use an SJA to advise (counsel) an individual about a social judgment. An individual is given a questionnaire similar to the one used in Step 6 of the knowledge mining process, and his/her responses are obtained for all the words in the vocabulary. These responses can then be compared with the outputs of the consensus SJA. If some or all of the individual's responses are "far" from those of the consensus SJA, then some action could

[5]The material in this paragraph is similar to Section 4.3.4 in [8].

be taken to sensitize the individual about these differences. More details about this are give in this section. Because of page limit, only a single-antecedent SJA is considered in this paper. Examples on multi-antecedent SJAs can be found in [14], [20].

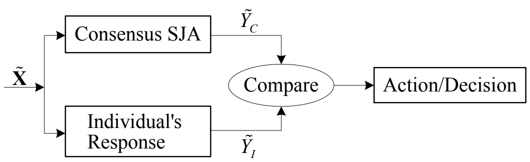

Fig. 5. One way to use the SJA for a social judgment.

The single-antecedent SJA describes the relationship between touching and flirtation and is denoted SJA_1. A consensus SJA_1 is constructed from Table IV, and is compared with an individual SJA.

When (11) is used to combine the different responses for each antecedent into a single consequent for the rule data in Table IV, one obtains the rule consequents depicted in Fig. 6(a). As a comparison, the rule consequents obtained from the original rule data in Table II are depicted in Fig. 6(b). Observe that:

1) The consequent for *none to very little* (NVL) touching is a left-shoulder in Fig. 6(a), whereas it is an interior FOU in Fig. 6(b). The former seems more reasonable to us.
2) The consequent for *some* (S) touching in Fig. 6(a) is similar to that in Fig. 6(b), except that it is shifted a little to the left. This is because the two largest responses [*large amount* (LA)] in Table II are removed in pre-processing.
3) The consequent for *moderate amount* (MOA) touching in Fig. 6(a) is similar to that in Fig. 6(b), except that it is shifted a little to the left. This is because the largest response [*maximum amount* (MAA)] in Table II is removed in pre-processing.
4) The consequent for *large amount* (LA) is similar to that in Fig. 6(b), except that it is shifted a little to the right. This is because the three smallest responses [*none to very little* (NVL)] in Table II are removed in pre-processing.
5) The consequent for *maximum amount* (MAA) is similar to those in Fig. 6(b), except that it is shifted a little to the right. This is because the three smallest responses [*none to very little* (NVL)] in Table II are removed in pre-processing.

The consequents $\tilde{Y}^1$–$\tilde{Y}^5$ shown in Fig. 6(a) are used in the rest of this section for the consensus SJA_1. Its five-rule rulebase is

R^1: IF touching is *NVL*, THEN flirtation is $\tilde{Y}^1$.
R^2: IF touching is *S*, THEN flirtation is $\tilde{Y}^2$.
R^3: IF touching is *MOA*, THEN flirtation is $\tilde{Y}^3$.
R^4: IF touching is *LA*, THEN flirtation is $\tilde{Y}^4$.
R^5: IF touching is *MAA*, THEN flirtation is $\tilde{Y}^5$.

For an input touching level, the output of SJA_1 can easily be computed by PR, as illustrated by the following:

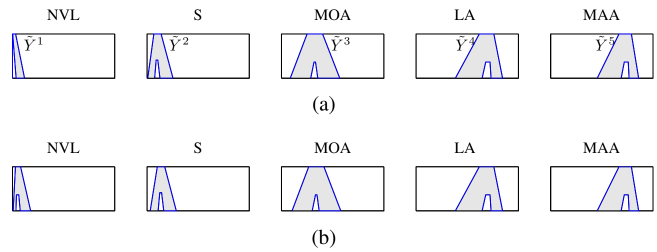

Fig. 6. Flirtation-level consequents of the five rules for the single-antecedent *touching* SJA_1: (a) with data pre-processing and (b) without data pre-processing. The level of touching is indicated at the top of each figure.

Example 2: Let observed touching be *somewhat small* (SS). The firing levels of the five rules are

$$f^1 = s_J(SS, NVL) = 0.08$$
$$f^2 = s_J(SS, S) = 0.43$$
$$f^3 = s_J(SS, MOA) = 0.12$$
$$f^4 = s_J(SS, LA) = 0$$
$$f^5 = s_J(SS, MAA) = 0$$

The resulting $\tilde{Y}_C$ computed from (13) is depicted in Fig. 7 as the dashed curve. The similarities between $\tilde{Y}_C$ and the nine words in the Fig. 3 vocabulary are computed to be:

$$\begin{array}{ll} s_J(\tilde{Y}_C, NVL) = .17 & s_J(\tilde{Y}_C, AB) = \mathbf{.67} \\ s_J(\tilde{Y}_C, SS) = .43 & s_J(\tilde{Y}_C, S) = .24 \\ s_J(\tilde{Y}_C, MOA) = .04 & s_J(\tilde{Y}_C, GA) = 0 \\ s_J(\tilde{Y}_C, CA) = 0 & s_J(\tilde{Y}_C, LA) = 0 \\ s_J(\tilde{Y}_C, MAA) = 0 & \end{array}$$

Because $\tilde{Y}_C$ and *AB* have the largest similarity, $\tilde{Y}_C$ is mapped into the word *AB*. □

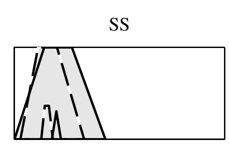

Fig. 7. $\tilde{Y}_C$ (dashed curve) and the mapped word (AB, solid curve) when touching is *somewhat small*.

When PR is used to combine the rules and any of the nine words in Fig. 3 are used as inputs, the outputs of the consensus SJA_1 are mapped to words shown in the second column of Table V. Each of these words was determined by using the same kind of calculations that were just described in Example 2. Observe that generally the flirtation level increases as touching increases, as one would expect.

Next, assume for the nine codebook words, an individual gives the responses[6] shown in the third column of Table V. Observe that this individual's responses are generally the

[6] The individual is asked the following question for each of the nine codebook words: *"If there is* (one of the nine codebook words) *touching, then what is the level of flirtation?"* and the answer must also be a word from the nine-word codebook.

same as or lower than $\tilde{Y}_C$. This means that this individual may under-react to touching.

The similarities between the consensus outputs $\tilde{Y}_C$ and the individual's responses $\tilde{Y}_I$, computed by using (5), are shown in the fourth column of Table V. $\tilde{Y}_I$ and $\tilde{Y}_C$ are said to be "significantly different" if $s_J(\tilde{Y}_C, \tilde{Y}_I)$ is smaller than a threshold θ. Let $\theta = 0.6$. Then, for the last four inputs, $\tilde{Y}_I$ and $\tilde{Y}_C$ are significantly different. Some action could be taken to sensitize the individual about these differences.

TABLE V

A COMPARISON BETWEEN THE CONSENSUS SJA_1 OUTPUTS AND AN INDIVIDUAL'S RESPONSES.

Touching	Flirtation level		$s_J(\tilde{Y}_C, \tilde{Y}_I)$
	$\tilde{Y}_C$	$\tilde{Y}_I$	
None to very little (NVL)	NVL	NVL	1
A bit (AB)	AB	AB	1
Somewhat small (SS)	AB	AB	1
Some (S)	SS	SS	1
Moderate amount (MOA)	SS	SS	1
Good amount (GA)	S	SS	0.12
Considerable amount (CA)	MOA	SS	0.56
Large amount (LA)	GA	SS	0.26
Maximum amount (MAA)	CA	MOA	0.21

IV. CONCLUSIONS

An application of Per-C to a social judgment has been introduced in this paper. First, a vocabulary is established for the social judgment and its words are modeled by IT2 FSs. Rule-consequent histograms are then obtained from surveys. Three pre-processing steps are used to remove bad responses and outliers. Because for each combination of inputs there may be several different consequents, PR is used to combine these consequents into a single one; hence, the rulebase is greatly simplified. PR is also used to infer the output FOU for input words that are not used in the survey. Finally, the output FOU is mapped back into a word in the codebook using the Jaccard similarity measure. So, from a user's point of view, he or she is interacting with the Per-C using only words from a vocabulary.

The techniques introduced in this paper should be applicable to many situations where rule-based decision-making is needed, and inputs and outputs are words.

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