FUZZ-IEEE 2009, Korea, August 20-24, 2009

Similarity-Based Perceptual Reasoning for Perceptual Computing

Dongrui Wu, *Student Member, IEEE*, and Jerry M. Mendel, *Life Fellow, IEEE*

***Abstract*— Perceptual reasoning (PR) is an approximate reasoning method that can be used as a computing with words (CWW) engine in perceptual computing. There can be different approaches to implement PR, e.g., PR using firing intervals is proposed in [8], [9], [16], and similarity-based PR is proposed in this paper. Both approaches satisfy the constraint on a CWW engine, i.e., the result of combining fired rules should lead to a footprint of uncertainty (FOU) that resembles the three kinds of FOUs in a CWW codebook. A comparative study shows that the output FOUs from similarity-based PR more closely resemble the three kinds of FOUs in a codebook, and the resulting linguistic descriptions are more intuitive; so, similarity-based PR is a better choice for a CWW engine.**

***Index Terms*— Computing with words, perceptual computing, perceptual reasoning, interval type-2 fuzzy sets, rule-based systems, similarity**

I. INTRODUCTION

The focus of this paper is on perceptual reasoning (PR) [8], [9], [16], a computing with words (CWW) engine [7] using IF-THEN rules. By a *rule*, we mean an IF-THEN statement, such as:

$$\begin{aligned} R^i: &\text{ IF } x_1 \text{ is } \tilde{F}_1^i \text{ and } x_2 \text{ is } \tilde{F}_2^i, \\ &\text{THEN } y \text{ is } \tilde{G}^i \qquad i = 1, ..., N \end{aligned} \tag{1}$$

where x_1 and x_2 are called *antecedents*, y is called *consequent*, and $\tilde{F}_1^i$, $\tilde{F}_2^i$ and $\tilde{G}^i$ are words modeled by interval type-2 fuzzy sets (IT2 FSs). A concrete example of such a two-antecedent rule is:

IF touching (x_1) is low amount $(\tilde{F}_1)$
and eye contact is (x_2) is moderate amount $(\tilde{F}_2)$,
THEN flirtation (y) is moderate amount $(\tilde{G})$.

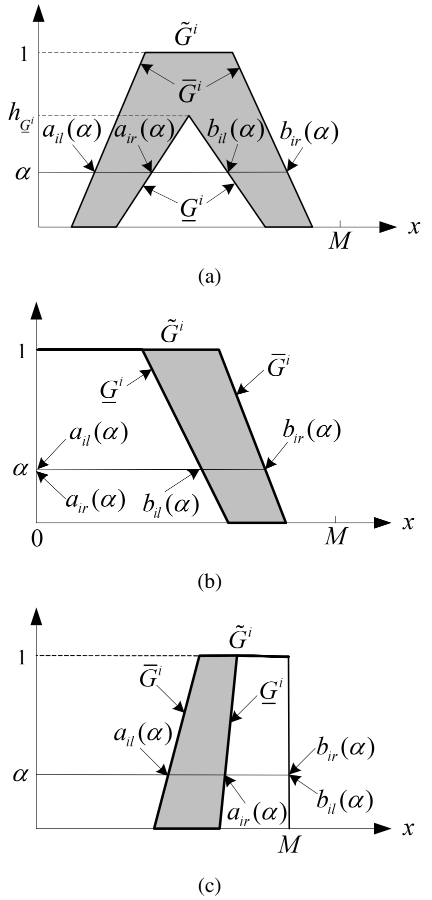
A generic rule with multiple antecedents is represented as:

$$\begin{aligned} R^i: &\text{ IF } x_1 \text{ is } \tilde{F}_1^i \text{ and } \cdots \text{ and } x_p \text{ is } \tilde{F}_p^i, \\ &\text{THEN } y \text{ is } \tilde{G}^i \qquad i = 1, ..., N \end{aligned} \tag{2}$$

The use of IF-THEN rules in a CWW engine is quite different from their use in most engineering applications of rule-based systems — fuzzy logic systems — because in a fuzzy logic system the output is almost always a number, whereas the output of the CWW engine is a footprint of uncertainty (FOU) which needs to be mapped into a recommendation.

According to the *Interval Approach* [3] for word modeling, only three kinds of word FOUs can be obtained: interior FOUs, left shoulders and right shoulders, as shown in Fig. 1. So, the following important requirement is imposed for PR [8], [9], [16]:

> *Requirement: The result of combining fired rules should lead to an FOU that resembles the three kinds of FOUs in a CWW codebook.*



(a)

(b)

(c)

Fig. 1. Typical word FOUs and an α-cut. (a) Interior FOU, (b) left-shoulder FOU, and (c) right-shoulder FOU.

PR using firing intervals has been proposed in [8], [9], [16]. It has been shown that PR using firing intervals satisfies the requirement; however, as will be shown in Section IV, generally the outputs of PR using firing intervals are much fatter than the codebook FOUs, and sometimes they are not intuitive. In this paper, similarity-based PR is proposed. It uses firing levels instead of firing intervals, and simulation results show that it is a better choice for a CWW engine.

The rest of this paper is organized as follows: Section II describes how firing intervals and firing levels are computed in PR. Section III provides properties about similarity-based PR. Section IV provides an example to show that similarity-

Dongrui Wu and Jerry M. Mendel are with the Signal and Image Processing Institute, Ming Hsieh Department of Electrical Engineering, University of Southern California, 3740 McClintock Ave., Los Angeles, CA 90089-2564. Email: dongruiw@usc.edu; mendel@sipi.usc.edu.

978-1-4244-3597-5/09/$25.00 ©2009 IEEE

based PR is a better approach for a CWW engine than PR using firing intervals. Finally, Section V draws conclusions.

II. PERCEPTUAL REASONING: COMPUTATIONS

PR [8], [9], [16] consists of two steps:

1) A *firing quantity* is computed for each rule, and
2) The IT2 FS consequents of the fired rules are combined using a linguistic weighted average (LWA) [12], [13] in which the "weights" are the firing quantities and the "signals" are the IT2 FS consequents.

Firing quantity calculations will be discussed in this section.

There can be two kinds of firing quantities — *firing interval* and *firing level*. Which firing quantity is better for PR is addressed in Section IV. The choice will be made so that the Requirement in the Introduction is better satisfied. Next, the mechanism of PR is explained.

Let $\tilde{\mathbf{X}}'$ denote an $N\times 1$ vector of IT2 FSs that are the inputs to a collection of N rules [see (2)], as would be the case when such inputs are words. $F^i(\tilde{\mathbf{X}}')$ denotes the firing quantity for the i^{th} rule, and it is computed only for the $n \le N$ number of fired rules, i.e., the rules whose firing quantities do not equal 0.

In PR, the fired rules are combined using a LWA. Denote the output IT2 FS of PR as $\tilde{Y}_{PR}$. Then, $\tilde{Y}_{PR}$ can be written in the following *expressive*[1] way:

$$\tilde{Y}_{PR} = \frac{\sum_{i=1}^{n} F^i(\tilde{\mathbf{X}}')\tilde{G}^i}{\sum_{i=1}^{n} F^i(\tilde{\mathbf{X}}')} \tag{3}$$

This LWA is a special case of the more general LWA [12], [13] in which both $\tilde{G}^i$ and $F^i(\tilde{\mathbf{X}}')$ are IT2 FSs. How to compute $\tilde{Y}_{PR}$ is explained next for two kinds of firing quantities, firing intervals and firing levels.

A. PR Using Firing Intervals

The result of the input and antecedent operations for the i^{th} fired rule can be a firing interval $F^i(\tilde{\mathbf{X}}')$, where [2], [4], [6]

$$F^i(\tilde{\mathbf{X}}') = [\underline{f}^i(\tilde{\mathbf{X}}'), \overline{f}^i(\tilde{\mathbf{X}}')] \equiv [\underline{f}^i, \overline{f}^i] \tag{4}$$

in which

$$\begin{aligned}\underline{f}^i(\tilde{\mathbf{X}}') = &\left[\sup_{x_1}\int_{x_1\in X_1} \underline{X}_1(x_1)\star \underline{F}_1^i(x_1)\right]\star\cdots \\ &\star\left[\sup_{x_p}\int_{x_p\in X_p} \underline{X}_p(x_p)\star \underline{F}_p^i(x_p)\right]\end{aligned} \tag{5}$$

$$\begin{aligned}\overline{f}^i(\tilde{\mathbf{X}}') = &\left[\sup_{x_1}\int_{x_1\in X_1} \overline{X}_1(x_1)\star \overline{F}_1^i(x_1)\right]\star\cdots \\ &\star\left[\sup_{x_p}\int_{x_p\in X_p} \overline{X}_p(x_p)\star \overline{F}_p^i(x_p)\right]\end{aligned} \tag{6}$$

and $\star$ denotes a t-norm. Though both minimum and product t-norms can be used in computing the firing intervals, for CWW the minimum t-norm is preferred for its simplicity. The detailed computations of (5) and (6) for typical word FOUs like the ones in Fig. 1 are presented in [9]. They are omitted here, because they are complicated, and, as is shown below in Section IV, PR using firing levels, rather than firing intervals, gives FOUs for $\tilde{Y}_{PR}$ that more closely resemble the FOUs of codebook words.

An interior FOU for rule consequent $\tilde{G}^i$ is depicted in Fig. 1(a), in which the height of $\underline{G}^i$ is denoted $h_{\underline{G}^i}$, the α-cut on $\underline{G}^i$ is denoted $[a_{ir}(\alpha), b_{il}(\alpha)]$, $\alpha \in [0, h_{\underline{G}^i}]$, and the α-cut on $\overline{G}^i$ is denoted $[a_{il}(\alpha), b_{ir}(\alpha)]$, $\alpha \in [0,1]$. For the left-shoulder $\tilde{G}^i$ depicted in Fig. 1(b), $h_{\underline{G}^i} = 1$ and $a_{il}(\alpha) = a_{ir}(\alpha) = 0$ for $\forall\alpha \in [0,1]$. For the right-shoulder $\tilde{G}^i$ depicted in Fig. 1(c), $h_{\underline{G}^i} = 1$ and $b_{il}(\alpha) = b_{ir}(\alpha) = M$ for $\forall\alpha \in [0,1]$.

Because the output of PR must resemble the three kinds of FOUs in a codebook, $\tilde{Y}_{PR}$ can also be an interior, left-shoulder or right-shoulder FOU, as shown in Fig. 2 (This is actually proved in [9], [16]). The α-cut on $\overline{Y}_{PR}$ is $[y_{Ll}(\alpha), y_{Rr}(\alpha)]$ and the α-cut on $\underline{Y}_{PR}$ is $[y_{Lr}(\alpha), y_{Rl}(\alpha)]$, where, as explained in [12], [13], the end-points of these α-cuts are computed as solutions to the following four optimization problems:

$$y_{Ll}(\alpha) = \min_{\forall f^i\in[\underline{f}^i,\overline{f}^i]} \frac{\sum_{i=1}^{n} a_{il}(\alpha) f^i}{\sum_{i=1}^{n} f^i}, \quad \alpha\in[0,1] \tag{7}$$

$$y_{Rr}(\alpha) = \max_{\forall f^i\in[\underline{f}^i,\overline{f}^i]} \frac{\sum_{i=1}^{n} b_{ir}(\alpha) f^i}{\sum_{i=1}^{n} f^i}, \quad \alpha\in[0,1] \tag{8}$$

$$y_{Lr}(\alpha) = \min_{\forall f^i\in[\underline{f}^i,\overline{f}^i]} \frac{\sum_{i=1}^{n} a_{ir}(\alpha) f^i}{\sum_{i=1}^{n} f^i}, \quad \alpha\in[0,h_{\underline{Y}_{PR}}] \tag{9}$$

$$y_{Rl}(\alpha) = \max_{\forall f^i\in[\underline{f}^i,\overline{f}^i]} \frac{\sum_{i=1}^{n} b_{il}(\alpha) f^i}{\sum_{i=1}^{n} f^i}, \quad \alpha\in[0,h_{\underline{Y}_{PR}}] \tag{10}$$

where

$$h_{\underline{Y}_{PR}} = \min_i h_{\underline{G}^i} \tag{11}$$

(7)-(10) can easily be computed by KM or EKM algorithms [6], [14], [15]. Details of the algorithms for computing $\tilde{Y}_{PR}$ can be found in [9].

Observe from (7) and (8) that $\overline{Y}_{PR}$, characterized by $[y_{Ll}(\alpha), y_{Rr}(\alpha)]$, is completely determined by $\overline{G}^i$, because it depends only on $a_{il}(\alpha)$ and $b_{ir}(\alpha)$, and from (9) and (10) that $\underline{Y}_{PR}$, characterized by $[y_{Lr}(\alpha), y_{Rl}(\alpha)]$, is completely determined by $\underline{G}^i$, because it depends only on $a_{ir}(\alpha)$ and $b_{il}(\alpha)$. Observe also, from (7) and (8), that $\tilde{Y}_{PR}$ is always normal, i.e., its $\alpha = 1$ α-cut can always be computed. This is different from many other Approximate Reasoning methods, whose aggregated fired-rule output sets are nor normal, e.g., the Mamdani-inference based method. For the latter, even if only one rule is fired, unless the firing interval is $[1,1]$, the output is a clipped or scaled version of the consequent IT2 FS instead of a normal IT2 FS [9]. This may cause problems when the output is mapped into a word in the codebook.

B. PR Using Firing Levels

Similarity is frequently used in Approximate Reasoning to compute the firing quantities [1], [10], [17], and it can also be

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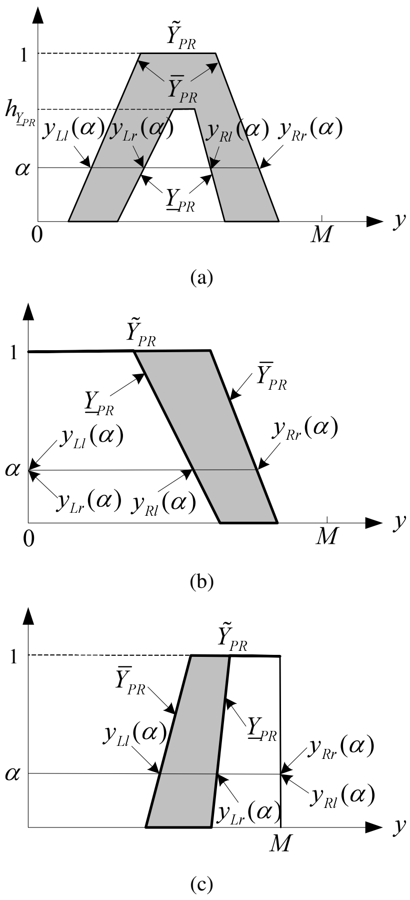
[1] We refer to (3) as "expressive" because it is not computed using multiplications, additions and divisions, as expressed by it. Instead, $\tilde{Y}_{PR}$ is computed using α-cuts, as explained later.



(a)

(b)

(c)

Fig. 2. PR FOUs and α-cuts on (a) interior, (b) left-shoulder, and (c) right-shoulder FOUs.

used in PR to compute the firing levels.

Let the p inputs that activate a collection of N rules be denoted $\tilde{\mathbf{X}}'$. The result of the input and antecedent operations for the i^{th} fired rule is the firing level $F^i(\tilde{\mathbf{X}}')$, where

$$F^i(\tilde{\mathbf{X}}') = s_J(\tilde{X}_1, \tilde{F}_1^i) \star \cdots \star s_J(\tilde{X}_p, \tilde{F}_p^i) \equiv f^i \tag{12}$$

in which $s_J(\tilde{X}_j, \tilde{F}_j^i)$ is the Jaccard similarity measure for IT2 FSs [11]:

$$s_J(\tilde{X}_j, \tilde{F}_j^i) = \frac{\int_X \min(\overline{X}_j(x), \overline{F}_j^i(x))dx + \int_X \min(\underline{X}_j(x), \underline{F}_j^i(x))dx}{\int_X \max(\overline{X}_j(x), \overline{F}_j^i(x))dx + \int_X \max(\underline{X}_j(x), \underline{F}_j^i(x))dx} \tag{13}$$

and $\star$ denotes a t-norm. The minimum t-norm is used in this paper.

The equations for computing $\overline{Y}_{PR}$ and $\underline{Y}_{PR}$ are similar to those described in Section II-A, except that now

$$\underline{f}^i = \overline{f}^i = f^i \tag{14}$$

Substituting (14) into (7)-(10), it follows that

$$y_{Ll}(\alpha) = \frac{\sum_{i=1}^n a_{il}(\alpha) f^i}{\sum_{i=1}^n f^i}, \quad \alpha \in [0, 1] \tag{15}$$

$$y_{Rr}(\alpha) = \frac{\sum_{i=1}^n b_{ir}(\alpha) f^i}{\sum_{i=1}^n f^i}, \quad \alpha \in [0, 1] \tag{16}$$

$$y_{Lr}(\alpha) = \frac{\sum_{i=1}^n a_{ir}(\alpha) f^i}{\sum_{i=1}^n f^i}, \quad \alpha \in [0, h_{\underline{Y}_{PR}}] \tag{17}$$

$$y_{Rl}(\alpha) = \frac{\sum_{i=1}^n b_{il}(\alpha) f^i}{\sum_{i=1}^n f^i}, \quad \alpha \in [0, h_{\underline{Y}_{PR}}] \tag{18}$$

Note that (15)-(18) are arithmetic weighted averages, so they are computed directly without using KM or EKM algorithms.

III. PERCEPTUAL REASONING: PROPERTIES

Properties of PR when firing intervals are used are given in [9], and this section only presents the properties of PR when firing levels are used. All of them help demonstrate the Requirement for PR, namely, *the result of combining fired rules using PR leads to an IT2 FS that resembles the three kinds of FOUs in a CWW codebook*. Only theorem statements are given in this paper due to space limit. Proofs of all theorems will be included in the journal version of this paper.

A. General Properties About the Shape of $\tilde{Y}_{PR}$

In this section, some general properties are provided that are about the shape of $\tilde{Y}_{PR}$. These general properties are used in Section III-B.

Theorem 1: When all fired rules have the same consequent $\tilde{G}$, $\tilde{Y}_{PR}$ defined in (3) is the same as $\tilde{G}$. ■

Although Theorem 6.1 is true regardless of how many rules are fired, its most interesting application occurs when only one rule is fired, in which case the output from PR is the consequent FS, $\tilde{G}$, where $\tilde{G}$ resides in the codebook. On the other hand, when one rule fires, the output from Mamdani inferencing is a clipped version of $\tilde{G}$, which does not reside in the codebook.

Theorem 2: $\tilde{Y}_{PR}$ is constrained by the consequents of the fired rules, i.e.,

$$\min_i a_{il}(\alpha) \le y_{Ll}(\alpha) \le \max_i a_{il}(\alpha) \tag{19}$$

$$\min_i a_{ir}(\alpha) \le y_{Lr}(\alpha) \le \max_i a_{ir}(\alpha) \tag{20}$$

$$\min_i b_{il}(\alpha) \le y_{Rl}(\alpha) \le \max_i b_{il}(\alpha) \tag{21}$$

$$\min_i b_{ir}(\alpha) \le y_{Rr}(\alpha) \le \max_i b_{ir}(\alpha) \tag{22}$$

where $a_{il}(\alpha)$, $a_{ir}(\alpha)$, $b_{il}(\alpha)$ and $b_{ir}(\alpha)$ are defined for three kinds of consequent FOUs in Fig. 1. ■

The equalities in (19)-(22) hold simultaneously if and only if all n fired rules have the same consequent. A graphical illustration of Theorem 2 is shown in Fig. 3. Assume only two rules are fired and $\tilde{G}^1$ lies to the left of $\tilde{G}^2$; then, $\tilde{Y}_{PR}$ lies between $\tilde{G}^1$ and $\tilde{G}^2$.

Theorem 2 is about the location of $\tilde{Y}_{PR}$. Theorem 3 below is about the span of $\tilde{Y}_{PR}$; but first, the span of an IT2 FS is defined.

Definition 1: The *span* of the IT2 FS $\tilde{G}^i$ is $b_{ir}(0) - a_{il}(0)$, where $a_{il}(0)$ and $b_{ir}(0)$ are the left and right end-points of the $\alpha = 0$ α-cut on $\tilde{G}^i$, respectively.

It is well-known from interval arithmetic that operations (e.g., $+$, $-$ and $\times$) on intervals usually spread out the resulting interval; however, this is not true for PR, as indicated by the following:

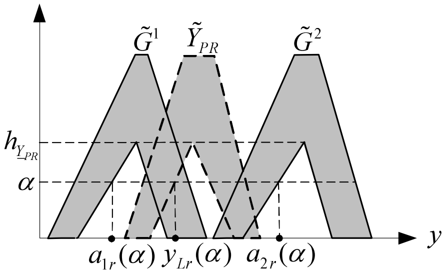


Fig. 3. A graphical illustration of Theorem 2, when only two rules fire.

Theorem 3: The span of $\tilde{Y}_{PR}$, $y_{Rr}(0) - y_{Ll}(0)$, is constrained by the spans of the consequents of the fired rules, i.e.,

$$\min_i (b_{ir}(0) - a_{il}(0)) \leq y_{Rr}(0) - y_{Ll}(0) \leq \max_i (b_{ir}(0) - a_{il}(0)). \quad \blacksquare \quad (23)$$

Both equalities in (23) hold simultaneously if and only if all n fired rules have the same span.

The following two definitions are about the shape of a T1 FS, and they are used in proving properties about the shape of $\tilde{Y}_{PR}$.

Definition 2: Let A be a type-1 FS and h_A be its height. Then, A is *trapezoidal-looking* if its $\alpha = h_A$ α-cut is an interval instead of a single point.

$\underline{Y}_{PR}$ and $\bar{Y}_{PR}$ in Fig. 2(a) are trapezoidal-looking.

Definition 3: Let A be a type-1 FS and h_A be its height. Then, A is *triangle-looking* if its $\alpha = h_A$ α-cut converges to a single point.

$\underline{Y}_{PR}$ in Fig. 3 is triangle-looking.

Theorem 4: Generally, $\underline{Y}_{PR}$ is trapezoidal-looking; however, $\underline{Y}_{PR}$ is triangle-looking if and only if all $\underline{G}_i$ are triangles with the same height. ■

Theorem 5: Generally, $\overline{Y}_{PR}$ is trapezoidal-looking; however, $\overline{Y}_{PR}$ is triangle-looking when all $\overline{G}^i$ are triangles. ■

B. The Geometry of $\tilde{Y}_{PR}$ FOUs

The following three lemmas are about the geometry of $\tilde{Y}_{PR}$ FOUs, and are useful in proving Theorems 6-8 in Section III-C:

Lemma 1: An IT2 FS $\tilde{Y}_{PR}$ is a *left shoulder FOU* [see Fig. 2(b)] if and only if $h_{\underline{Y}_{PR}} = 1$ and $y_{Lr}(1) = 0$. ■

Lemma 2: An IT2 FS $\tilde{Y}_{PR}$ is a *right shoulder FOU* [see Fig. 2(c)] if and only if $h_{\underline{Y}_{PR}} = 1$ and $y_{Rl}(1) = M$. ■

Lemma 3: An IT2 FS $\tilde{Y}_{PR}$ is an *interior FOU* [see Fig. 2(a)] if and only if:

(1) $h_{\underline{Y}_{PR}} < 1$; or
(2) $h_{\underline{Y}_{PR}} = 1$, $y_{Lr}(1) > 0$ and $y_{Rl}(1) < M$. ■

C. Properties of $\tilde{Y}_{PR}$ FOUs

In this subsection it is shown that $\tilde{Y}_{PR}$ computed from (3), that uses firing levels, resembles the three kinds of FOUs in a CWW codebook.

Theorem 6: Let $\tilde{Y}_{PR}$ be expressed as in (3). Then, $\tilde{Y}_{PR}$ is a *left shoulder FOU* if and only if all $\tilde{G}^i$ are left shoulder FOUs. ■

Theorem 7: Let $\tilde{Y}_{PR}$ be expressed as in (3). Then, $\tilde{Y}_{PR}$ is a *right shoulder FOU* if and only if all $\tilde{G}^i$ are right shoulder FOUs. ■

Theorem 8: Let $\tilde{Y}_{PR}$ be expressed as in (3). Then, $\tilde{Y}_{PR}$ is an *interior FOU* if at least one $\tilde{G}^i$ is an interior FOU. ■

Theorems 6-8 are important because they show that the output of PR is a normal IT2 FS and is similar to the word FOUs in a codebook[2]. So, the Jaccard similarity measure [11] can be used to map $\tilde{Y}_{PR}$ to a word in the codebook. On the other hand, it is less intuitive to map a clipped FOU, as obtained from a Mamdani inference mechanism, or a crisp point, as obtained from the TSK inference mechanism, to a normal IT2 FS word FOU in the codebook.

IV. Comparative Study

Since the outputs of both PR approaches resemble the three kinds of word FOUs, a natural question to ask is: Which approach is better? In this section an example is used to compare the outputs from the two PR approaches. The criteria for determining a better PR approach are:

1) The PR outputs should closely resemble the three kinds of word FOUs, and the closer the resemblance the better.
2) The PR outputs should be intuitive, i.e., they should coincide with our expectations.

A few of the rules from a Social Judgment Advisor [5] are used in our comparative study. For notation simplicity, the output IT2 FS from PR using firing intervals is denoted as $\tilde{Y}^I_{PR}$, and the output IT2 FS from similarity-based PR is denoted as $\tilde{Y}^S_{PR}$. A 5-word vocabulary for inputs and outputs is depicted in Fig. 4; it is a subset of a 30-word vocabulary obtained from the Interval Approach in [3]. Four rules, from a set of 25 rules, for the social judgment of flirtation are:

R^1: IF touching is *Moderate amount* and eye contact is *Moderate amount*, THEN flirtation is $\tilde{Y}^1$.
R^2: IF touching is *Moderate amount* and eye contact is *Large amount*, THEN flirtation is $\tilde{Y}^2$.
R^3: IF touching is *Large amount* and eye contact is *Moderate amount*, THEN flirtation is $\tilde{Y}^3$.
R^4: IF touching is *Large amount* and eye contact is *Large amount*, THEN flirtation is $\tilde{Y}^4$.

FOUs for $\tilde{Y}^i$ are depicted in Fig. 5. They were computed from survey histograms [6], and the centers of their centroids are in Table I. Observe that as either touching or eye contact increases, the flirtation consequent also increases, i.e., the rules implement a monotonic mapping, as one would expect for the social judgment of flirtation.

Modest amount and *A lot*, whose FOUs are depicted in Fig. 4, are used to excite the four rules in this comparative study.

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[2] A small difference is that the lower MFs of interior codebook word FOUs are always triangular, whereas the lower MFs of interior $\tilde{Y}_{PR}$ are usually trapezoidal.

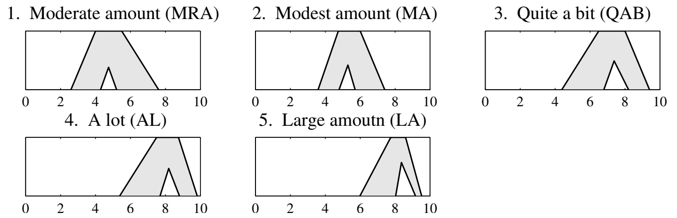
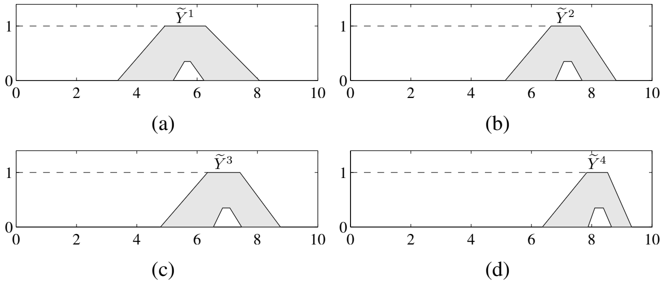


Fig. 4. Five word FOUs ranked by their centers of centroid.



(a) (b)

(c) (d)

Fig. 5. Consequent FOUs for the four rules.

A. Results for PR Using Firing Intervals

Four combinations of inputs were considered for PR using firing intervals, as shown in the first two columns of Table II. The PR FOUs are shown in Column 3, and each of them is mapped into a word in the 5-word vocabulary using the Jaccard similarity measure [11]. Their centers of centroids and similarities to the five words are also given in Table II.

Recall that to map $\tilde{Y}^I_{PR}$ into a word in the 5-word vocabulary, the similarities between $\tilde{Y}^I_{PR}$ and the five words in Fig. 4 are computed and then the word with the maximum similarity is chosen. Because in this simple example a linguistic level of flirtation must be provided using just the Fig. 4 5-word vocabulary, one of those words must be chosen; so, even though $\tilde{Y}^I_{PR}$ (the dashed FOU in the first row of Table II) and $FOU(QAB)$ do not look so similar, "*Quite a bit (of flirtation)*" is that word. When firing intervals are used, all four pairs of linguistic inputs lead to the same level of flirtation — "*Quite a bit*."

B. Results for Similarity-Based PR

The same four combinations of touching and eye contact are considered for similarity-based PR. The corresponding results are given in Table III. Now all four pairs of linguistic inputs do not lead to the same level of flirtation.

C. Observations

The following observations can be made from the above results:

1) Observe from the third column of Tables II and III that: a) $\tilde{Y}^I_{PR}$ and $\tilde{Y}^S_{PR}$ are different, and $\tilde{Y}^I_{PR}$ is fatter than both $\tilde{Y}^S_{PR}$ and the word FOUs in the 5-word vocabulary; b) For the same levels of touching and eye contact, $\tilde{Y}^I_{PR}$ and $\tilde{Y}^S_{PR}$ may be mapped into different words in the 5-word vocabulary; and, c) The four $\tilde{Y}^I_{PR}$ for the four combinations of touching and eye contact are mapped into the same word "*Quite a bit*," which is counter-intuitive because the linguistic description of flirtation level is expected to change as inputs change; on the other hand, similarity-based PR gives more intuitive linguistic flirtation levels.

TABLE I

CENTERS OF CENTROID OF THE RULE CONSEQUENTS $\tilde{Y}^i$, $i = 1, 2, 3, 4$.

Consequent	$\tilde{Y}^1$	$\tilde{Y}^2$	$\tilde{Y}^3$	$\tilde{Y}^4$
Center of Centroid	5.6913	7.0804	6.8625	8.0446

2) Observe from the fourth column of Tables II and III that the centers of centroid of both $\tilde{Y}^I_{PR}$ and $\tilde{Y}^S_{PR}$ increase monotonically as either input increases, which is consistent with the rulebase and our expectation.
3) Observe from the second row of Tables II and III that when the two PR approaches give the same linguistic flirtation level (*Quite a bit*), $s_J(\tilde{Y}^S_{PR}, QAB) > s_J(\tilde{Y}^I_{PR}, QAB)$ and *Quite a bit* is more dominant for similarity-based PR than PR using firing intervals (e.g., the difference between $s_J(\tilde{Y}^S_{PR}, QAB)$ and any other similarity in the second row of Table III is larger than its corresponding difference in the second row of Table II), i.e., one is more confident about the linguistic flirtation levels obtained from similarity-based PR. The same conclusion can be drawn from the third row of Tables II and III.

In summary, the output FOUs from similarity-based PR more closely resemble the codebook FOUs, and the resulting linguistic flirtation levels are more intuitive; so, similarity-based PR is a better choice as a CWW engine than PR using firing intervals.

V. CONCLUSIONS

In this paper, perceptual reasoning, which is an approximate reasoning method, has been reviewed. What differentiates PR from other approximate reasoning methods is the shape of its output FOU, i.e., the output FOU of PR resembles the three kinds of FOUs in a CWW codebook whereas other approximate reasoning methods cannot achieve this. A similarity-based perceptual reasoning approach has also been introduced, and it is compared with PR using firing intervals. A comparative study showed that the output FOUs from similarity-based PR more closely resemble the three kinds of FOUs in a codebook, and the resulting linguistic flirtation levels are more intuitive; so, our preference is PR using similarity for a CWW engine.

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TABLE II

COMPARATIVE STUDY RESULTS FOR PR USING FIRING INTERVALS.

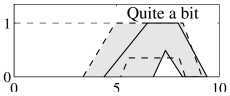
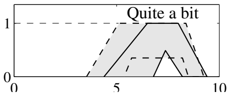
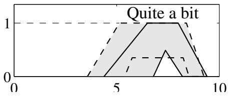
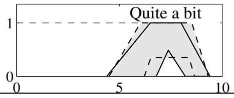
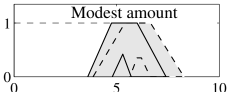
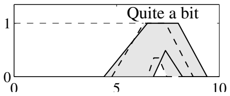
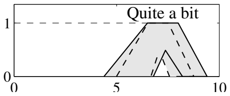
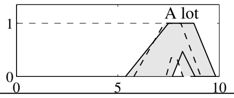
Touching	Eye contact	$\tilde{Y}^I_{PR}$ (dashed) and the mapped word (solid)	Center of centroid	$s_J(\tilde{Y}^I_{PR}, MRA)$	$s_J(\tilde{Y}^I_{PR}, MA)$	$s_J(\tilde{Y}^I_{PR}, QAB)$	$s_J(\tilde{Y}^I_{PR}, AL)$	$s_J(\tilde{Y}^I_{PR}, LA)$
Modest amount	Modest amount		6.5480	0.3680	0.4483	**0.6258**	0.3778	0.3021
A lot	Modest amount		6.6929	0.3366	0.4173	**0.6435**	0.4191	0.3386
Modest amount	A lot		6.7210	0.3299	0.4091	**0.6468**	0.4272	0.3461
A lot	A lot		7.2068	0.2053	0.2536	**0.7993**	0.5620	0.4633

TABLE III

COMPARATIVE STUDY RESULTS FOR SIMILARITY-BASED PR.

Touching	Eye contact	$\tilde{Y}^S_{PR}$ (dashed) and the mapped word (solid)	Center of centroid	$s_J(\tilde{Y}^S_{PR}, MRA)$	$s_J(\tilde{Y}^S_{PR}, MA)$	$s_J(\tilde{Y}^S_{PR}, QAB)$	$s_J(\tilde{Y}^S_{PR}, AL)$	$s_J(\tilde{Y}^S_{PR}, LA)$
Modest amount	Modest amount		6.0755	0.4244	**0.5621**	0.4369	0.2225	0.1624
A lot	Modest amount		6.8326	0.2194	0.2762	**0.6869**	0.3847	0.3019
Modest amount	A lot		6.9761	0.1864	0.2304	**0.6852**	0.4248	0.3356
A lot	A lot		7.6026	0.0881	0.0990	0.5952	**0.6403**	0.5789

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