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Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers

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Abstract

Type-2 fuzzy sets, which are characterized by membership functions (MFs) that are themselves fuzzy, have been attracting interest. This paper focuses on advancing the understanding of interval type-2 fuzzy logic controllers (FLCs). First, a type-2 FLC is evolved using Genetic Algorithms (GAs). The type-2 FLC is then compared with another three GA evolved type-1 FLCs that have different design parameters. The objective is to examine the amount by which the extra degrees of freedom provided by antecedent type-2 fuzzy sets is able to improve the control performance. Experimental results show that better control can be achieved using a type-2 FLC with fewer fuzzy sets/rules so one benefit of type-2 FLC is a lower trade-off between modeling accuracy and interpretability.

Keywords: Type-2 fuzzy logic controller; Genetic algorithms; Process control; Modelling uncertainty

1. Introduction

Fuzzy logic was introduced by Lotfi Zadeh for emulating a human's ability to reason and solve problems using imprecise information. Its underlying modes of reasoning are approximate. Fuzzy logic systems (FLSs) are generally knowledge-based systems consisting of linguistic "If–Then" rules that can be constructed using the knowledge of experts in the given field of interest. Fig. 1 shows a rule-based FLS. The fuzzifier, inference mechanism (which is associated with the expert rules, the heart of an FLS), and defuzzifier involve operations on fuzzy sets that are defined by membership functions (MFs). When FLSs are used for control, they are called fuzzy logic controllers (FLCs). FLCs have demonstrated their ability in a number of applications (John and Langari, 1998), especially for the control of complex non-linear systems that may be difficult to model analytically (King and Mamdani, 1977; Umbers and King, 1980).

Researches have shown that there may be limitations in the ability of type-1 FLSs to model and minimize the effect of uncertainties (Mendel, 2001; Hagras, 2004). One constrain being that a type-1 fuzzy set is certain in the sense that the membership grade for each input is a crisp value. Recently, type-2 fuzzy sets, characterized by MFs that are themselves fuzzy, have been attracting interest (Mendel, 2001). The key concept is the footprint of uncertainty (FOU), which models the uncertainties in the shape and position of the type-1 fuzzy set. Fig. 2 illustrates two type-2 fuzzy MFs with FOUs shown as the shaded areas. They are obtained by blurring a type-1 Gaussian MF with mean m and deviation δ. In Fig. 2(a), the type-2 Gaussian MF is derived by keeping the mean m constant and allowing the deviation to vary between δ_1 and δ_2. Conversely, Fig. 2(b) is generated when the mean of the Gaussian function assumes values in the range $[m_1, m_2]$ while the deviation is maintained at δ. Interval type-2 fuzzy sets (Liang and Mendel, 2000) are the most common. For such sets, each point in the FOU has unity secondary membership grade. They may be uniquely defined by the two type-1 MFs, upper MF and lower MF, that bound the FOU.

A FLS described using at least one type-2 fuzzy set is called a type-2 FLS. Type-2 FLSs have been used successfully in many applications, for example, time-series forecasting (Mendel, 2001), communication and networks

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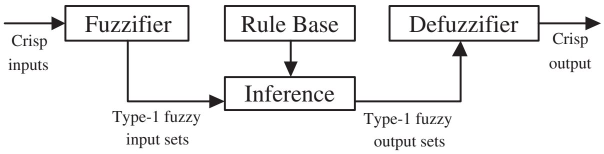

Fig. 1. A type-1 FLS.

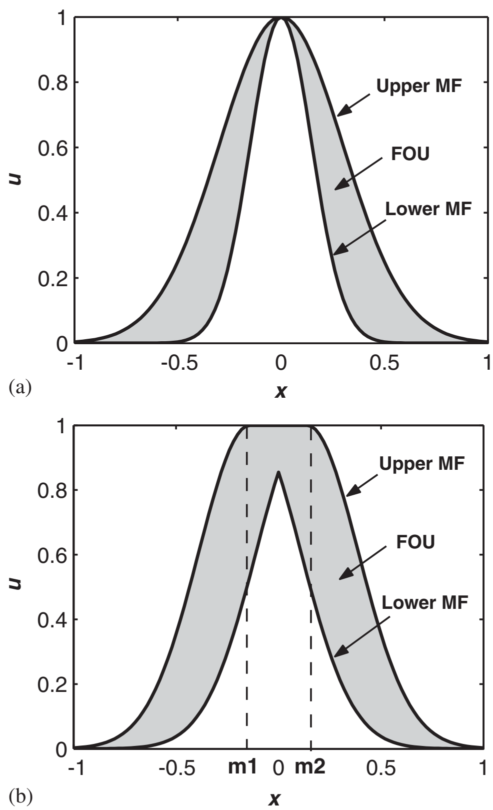

Fig. 2. Interval type-2 fuzzy sets: (a) An interval type-2 fuzzy set with uncertain deviation; (b) an interval type-2 fuzzy set with uncertain mean.

(Liang and Mendel, 2001), decision making (Ozen et al., 2004), data and survey processing (John et al., 2000; Mendel, 2001), word modeling (Wu and Mendel, 2004) and phoneme recognition (Zeng and Liu, 2004). Even though fuzzy control is the most widely used application of fuzzy set theory, a literature search reveals that only a few type-2 FLSs are employed in the field of control. Interval type-2 FLCs were applied to mobile robot control (Hagras, 2004), quality control of sound speakers (Melin and Castillo, 2002), connection admission control in ATM networks (Liang et al., 2000). A dynamical optimal training algorithm for type-2 fuzzy neural networks (T2FNNs) has also been proposed (Wang et al., 2004). T2FNNs have been used for truck back up control (Wang et al., 2004). The wide range of successful applications is an indication that type-2 FLSs may provide good solutions, especially in the presence of uncertainties.

This paper is a contribution towards the development and understanding of type-2 fuzzy control. A genetic learning strategy for designing a type-2 FLS to control non-linear plants is proposed. GA, a global optimal search algorithm, has been widely used to design FLSs (Wang et al., 2004). Due to the computational requirements, FLCs designed using GA are generally evolved off-line using a model of the controlled process. As it is impossible for a model to capture all the characteristics of the actual plant, the performance of the type-1 FLC designed using GA and a theoretical model will inevitably deteriorate when it is applied to the real-world problem. The concept of type-2 fuzzy sets was introduced to enhance the uncertainty handling capability of FLS. An issue that is addressed herein is whether a FLC that utilizes antecedent type-2 fuzzy sets would cope better with modeling uncertainties, and thereby achieve better control performance than a type-1 FLC in practice. The study is performed by comparing the ability of type-1 and type-2 FLCs to control an uncertain liquid level plant. One aspect that was considered in the comparative study is the number of design parameters or degrees of freedom that the FLCs have. It is well-known that the performance of a type-1 FLC may be improved by partitioning the input domains with a larger number of fuzzy sets. Unfortunately, there is a trade-off between accuracy/performance and interpretability. A larger number of MFs results in a bigger rule base that would be harder for a human to interpret because of the curse of dimensionality. Since the FOU provides a type-2 fuzzy set with an additional mathematical dimension, the conjecture is that a type-2 FLC with a smaller rule base may be capable of providing performance comparable to a type-1 FLC that has more rules. Hence, another objective is to ascertain whether a type-2 FLC is able to provide better performance/accuracy without sacrificing rule base interpretability.

The rest of the paper is organized as follows: Section 2 describes the interval singleton type-2 FLC used in this work. GAs and approaches for designing type-2 FLCs are briefly introduced in Section 3. Next, details of the FLCs that were evolved by GA are covered in Section 4. Section 5 presents the comparative abilities of the FLCs to handle modelling uncertainties. Discussions are given in Section 6 before conclusions are drawn in Section 7.

2. Interval singleton type-2 FLC

The structure of a type-2 FLS is shown in Fig. 3. It is similar to its type-1 counterpart, the major difference being that at least one of the fuzzy sets in the rule base is type-2. Hence, the output of the inference engine are type-2 sets and a type-reducer is needed to convert them into type-1 sets before defuzzification can be carried out.

2.1. Inference

In this paper, an interval singleton type-2 FLS (Mendel, 2001) is employed. "Interval" means that the input/output domains are characterized by interval type-2 sets (Liang

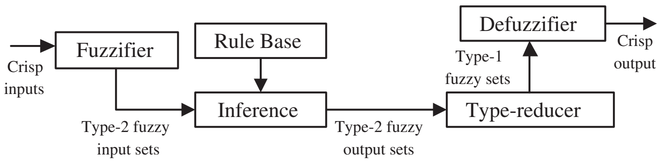

Fig. 3. A type-2 FLS.

and Mendel, 2000), whereby the membership grades of all elements in the FOU (*secondary membership grades*) are unity. The term "singleton" denotes that the fuzzifier converts the input signals of the FLC into fuzzy singletons. The inference engine then matches the fuzzy singletons with the fuzzy rules in the rule base. To compute unions and intersections of type-2 sets, compositions of type-2 relations are needed. Just as the sup-star composition is the backbone computation for a type-1 FLC, the extended sup-star composition is the backbone for a type-2 FLC (Mendel, 2001). To illustrate the extended sup-star operation, consider a rule base that consists of rules with the following structure:

R^i: If x_1 is $\tilde{F}_1^i$ and x_2 is $\tilde{F}_2^i$, then y is $\tilde{G}^i$,

where $i = 1, 2, \ldots, N$ and N is the number of rules. The first step in the extended sup-star operation is to obtain the firing set $\sqcap_{j=1}^{2} \mu_{\tilde{F}_j^i}(x_j) \equiv F^i(\mathbf{x})$ by performing the input and antecedent operations. As only interval type-2 sets are used and the meet operation is implemented by the product t-norm, the firing set is the following type-1 interval set:

$$F^i(\mathbf{x}) = [\underline{f}^i(\mathbf{x}),\ \overline{f}^i(\mathbf{x})] \equiv [\underline{f}^i,\ \overline{f}^i], \tag{1}$$

where $\underline{f}^i(\mathbf{x}) = \underline{\mu}_{\tilde{F}_1^i}(x_1) \cdot \underline{\mu}_{\tilde{F}_2^i}(x_2)$ and $\overline{f}^i(\mathbf{x}) = \overline{\mu}_{\tilde{F}_1^i}(x_1) \cdot \overline{\mu}_{\tilde{F}_2^i}(x_2)$. The term $\underline{\mu}_{\tilde{F}_j^i}(x_j)$ and $\overline{\mu}_{\tilde{F}_j^i}(x_j)$ are the lower and upper membership grades of $\mu_{\tilde{F}_j^i}(x_j)$ (see Fig. 2). Next, the firing set, $F^i(\mathbf{x})$, is combined with the consequent fuzzy set of the ith rule using the product t-norm to derive the fired output consequent sets. The combined output fuzzy set may then be obtained using the maximum t-conorm.

2.2. Type-reduction and defuzzification

Since the output of the inference engine is a type-2 fuzzy set, it must be type-reduced before the defuzzifier can be used to generate a crisp output. This is the main structural difference between type-1 and type-2 FLCs. The most commonly used type-reduction method is the center-of-sets type-reducer, which may be expressed as (Mendel, 2001)

$$Y_{\cos}(\mathbf{x}) = \int_{y^1 \in Y^i} \cdots \int_{y^N \in Y^i} \int_{f^1 \in F^i(\mathbf{X})} \cdots \int_{f^N \in F^i(\mathbf{X})} 1 \Bigg/ \frac{\sum_{i=1}^{N} f^i y^i}{\sum_{i=1}^{N} f^i} = [y_l,\ y_r], \tag{2}$$

where $F^i(\mathbf{X}) = [\underline{f}^i, \overline{f}^i]$ is the interval firing level of the ith rule, $Y^i = [y_l^i, y_r^i]$ is an interval type-1 set corresponding to the centroid of the interval type-2 consequent set $\tilde{G}^i$ (Mendel, 2001)

$$C_{\tilde{G}^i} = \int_{\theta_1 \in J_{x_1}} \cdots \int_{\theta_N \in J_{x_N}} 1 \Bigg/ \frac{\sum_{i=1}^{N} x_i \theta_i}{\sum_{i=1}^{N} \theta_i} = [y_l^i,\ y_r^i]. \tag{3}$$

Eq. (2) may be computed using the Karnik–Mendel iterative method (Mendel, 2001) as follows:

Set $y^i = y_l^i$ (*or* y_r^i) *for* $i = 1, \ldots, N$;
Arrange y^i *in ascending order*;
Set $f^i = \dfrac{\underline{f}^i + \overline{f}^i}{2}$ *for* $i = 1, \ldots, N$;
$y' = \dfrac{\sum_{i=1}^{N} y^i f^i}{\sum_{i=1}^{N} f^i}$;
do
  $y'' = y'$;
  Find $k \in [1, N-1]$ *such that* $y^k \leqslant y' \leqslant y^{k+1}$;
  Set $f^i = \overline{f}^i$ (*or* $\underline{f}^i$) *for* $i \leqslant k$
  Set $f^i = \underline{f}^i$ (*or* $\overline{f}^i$) *for* $i \geqslant k+1$;
  $y' = \dfrac{\sum_{i=1}^{N} y^i f^i}{\sum_{i=1}^{N} f^i}$;
while $y' \neq y''$
y_l(*or* y_r) $= y'$;

It has been proven that this iterative procedure can converge in at most N iterations (Mendel, 2001). Once y_l and y_r are obtained, they can be used to calculate the crisp output. Since the type-reduced set is an interval type-1 set, the defuzzified output is:

$$y(\mathbf{x}) = \frac{y_l + y_r}{2}. \tag{4}$$

3. Genetic learning of a type-2 FLC

GA is a general-purpose search algorithm that uses principles inspired by natural population genetics to evolve solutions to problems. It was first proposed in 1975 (Holland, 1975). GAs are theoretically and empirically proven to provide a robust search in complex spaces, thereby offering a valid approach to problems requiring efficient and effective searches (Goldberg, 1989; Sakawa, 2002).

Fig. 4 contains the flow chart of a basic GA. First, a chromosome population is randomly generated. Each chromosome encodes a candidate solution of the optimization problem. The fitness of all individuals with respect to the optimization task is then evaluated by a scalar objective function (fitness function). According to Darwin's principle, highly fit individuals are more likely to be selected to reproduce offspring. Genetic operators such as crossover and mutation are applied to the parents in order to produce a new generation of candidate

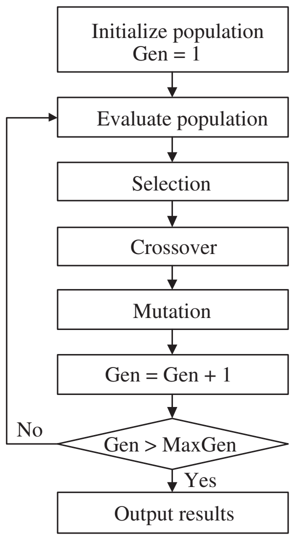

Fig. 4. The flow chart of a basic GA.

solutions. As a result of this evolutionary cycle of selection, crossover and mutation, more and more suitable solutions to the optimization problem emerge within the population.

Increasingly, GA is used to facilitate FLSs design (Cordon et al., 2001, 2004; Homaifar and McCormick, 1995; Kim et al., 1995). However, most of the works discuss type-1 FLC design. This paper focuses on genetic learning of type-2 FLCs. There are two very different approaches for selecting the parameters of a type-2 FLS (Mendel, 2001). One is the partially dependent approach, where a best possible type-1 FLS is designed first, and then used to initialize the parameters of a type-2 FLS. The other method is a totally independent approach, where all the parameters of the type-2 FLS are tuned from scratch without the aid of an existing type-1 design.

One advantage offered by the partially dependent approach is smart initialization of the parameters of the type-2 FLS. Since the baseline type-1 fuzzy sets impose constraints on the type-2 sets, fewer parameters need to be tuned and the search space for each variable is smaller. Therefore, the computational cost needed to implement the GA is less than that of the totally independent approach. So design flexibility is traded for a lower computational burden. Type-2 FLCs designed via the partially dependent approach are able to outperform the corresponding type-1 FLCs (Wu and Tan, 2004), although both the FLCs have the same number of MFs (resolution). However, the type-2 FLC has a larger number of degrees of freedom because the fuzzy set is more complex. The additional mathematical dimension provided by the type-2 fuzzy set enables a type-2 FLS to produce more complex input–output map without the need to increase the resolution. An open question is whether a type-1 FLS with a higher resolution, and therefore more degrees of freedom, would be able to match the modeling capability of a type-2 FLS. To address this issue, a comparative study involving type-2 and type-1 FLCs with similar number of degrees of freedom is performed. The totally independent approach is adopted so that the type-2 FLC evolved using GA has maximum design flexibility. Details about the FLCs are delineated in the following section.

4. Structure of FLCs used in comparative studies

Four two-inputs single-output FLCs with different design parameters are studied. The input signals of all the FLCs are the feedback error, e, and the change of the error, $\dot{e}$, and the output signal is the change in the control signal, $\dot{u}$.

4.1. The type-2 FLC, FLC_2

Each input domain of FLC_2 is partitioned by three interval type-2 fuzzy sets (Gaussian MFs with constant mean and uncertain variance) that are labeled as **N**, **Z** and **P** (refer to Fig. 8(a)). In order to study the benefits of antecedent type-2 fuzzy sets, its effect is isolated by using five crisp numbers $\dot{u}_i$ $(i = 1, 2, \ldots, 5)$ as the consequents. Table 1 shows the fuzzy rule base used by the type-2 FLC. As the GA will only tune the MFs, the rules are fixed so a commonly used rule base is employed.

Fig. 2(a) shows that a Gaussian MF with certain mean and uncertain variance can be completely defined by 3 parameters, m and $[\delta_1, \delta_2]$. The MFs of $\dot{u}$ are completely described by five distinct numbers (points). As FLC_2 has 6 input type-2 MFs and 5 different crisp outputs, FLC_2 has a total of $3 \times 6 + 5 = 23$ parameters. A chromosome is shown in Fig. 5.

Table 1
Rule base of FLC_2 and FLC_{1a}

e	$\dot{e}$		
	$N_{\dot{e}}$	$Z_{\dot{e}}$	$P_{\dot{e}}$
N_e	$\dot{u}_1$	$\dot{u}_2$	$\dot{u}_3$
Z_e	$\dot{u}_2$	$\dot{u}_3$	$\dot{u}_4$
P_e	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_5$

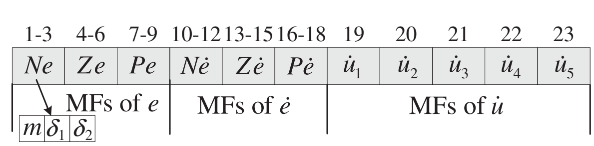

Fig. 5. GA coding scheme of FLC_2.

Table 2
Rule base of the type-1 FLC, FLC_{1b}

e	$\dot{e}$				
	$NB_{\dot{e}}$	$NM_{\dot{e}}$	$Z_{\dot{e}}$	$PM_{\dot{e}}$	$PB_{\dot{e}}$
NB_e	$\dot{u}_1$	$\dot{u}_2$	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_5$
NM_e	$\dot{u}_2$	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_5$	$\dot{u}_6$
Z_e	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_5$	$\dot{u}_6$	$\dot{u}_7$
PM_e	$\dot{u}_4$	$\dot{u}_5$	$\dot{u}_6$	$\dot{u}_7$	$\dot{u}_8$
PB_e	$\dot{u}_5$	$\dot{u}_6$	$\dot{u}_7$	$\dot{u}_8$	$\dot{u}_9$

4.2. The type-1 FLC, FLC_{1a}

The structure and rule base of the type-1 FLC, FLC_{1a}, are the same as those of FLC_2. The only difference between FLC_{1a} and FLC_2 is that the input MFs of FLC_{1a} are type-1 (refer to Fig. 8(b)). Product-sum inference and height defuzzification were employed. Since two parameters are sufficient to determine a Gaussian type-1 MF, the GA has to optimize a total of $2 \times 6 + 5 = 17$ parameters. FLC_2 and FLC_{1a} have the same number of MFs and rules. Hence, comparing their performances may provide insights into the contributions made by the FOU.

4.3. The type-1 FLC, FLC_{1b}

Each input of FLC_{1b} has 5 type-1 MFs in its universe of discourse, as shown in Fig. 8(c). The rule base is given in Table 2. It is commonly used by Mamdani FLCs. FLC_{1b} has $2 \times 10 + 9 = 29$ parameters to be tuned. Compared to FLC_2, FLC_{1b} has 6 extra design parameters. They enable us to determine whether a type-2 FLC is able to outperform a type-1 FLC with similar number of degrees of freedom.

4.4. The neuro-fuzzy controller, NFC

The fourth controller analyzed in this paper is a neuro-fuzzy controller similar to the one used by Teo et al. (1998). Each of its two inputs is characterized by 5 type-1 MFs, as shown in Fig. 8(d). Though the input MFs are similar to those of FLC_{1b}, its rule base is quite different. The consequents of the 25 rules are different from each other (refer to Table 7(b)). Thus, there are $2 \times 10 + 25 = 45$ parameters to be tuned by GA.

5. Experimental comparison of type-1 and type-2 FLCs

This section presents an experimental comparison of the characteristics of the four FLCs. The test platform is a non-linear second order liquid level process. Since the FLCs are tuned offline, the simulation model used for identifying the controller parameters is described in the following subsection.

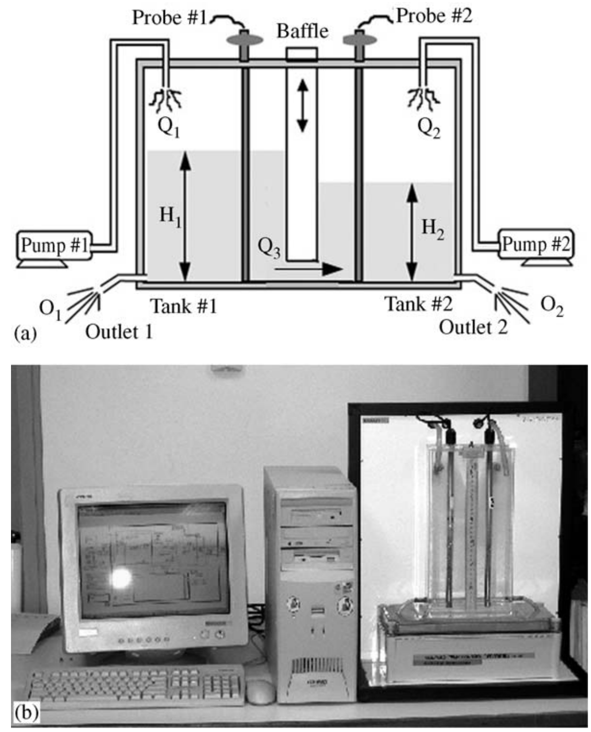

Fig. 6. The coupled-tank liquid-level control system: (a) Schematic diagram; (b) experimental setup.

5.1. The coupled-tank system

The coupled-tank apparatus shown in Fig. 6 is used to assess the FLCs. It consists of two small tower-type tanks mounted above a reservoir that stores the water. Water is pumped into the top of each tank by two independent pumps, and the levels of water are measured by two capacitive-type probe sensors. Each tank is fitted with an outlet, at the side near the base. Raising the baffle between the two tanks allows water to flow between them. The amount of water that returns to the reservoir is approximately proportional to the square root of the height of water in the tank, which is the main source of non-linearity in the system.

The dynamics of the coupled-tank apparatus can be described by the following set of non-linear differential equations:

$$A_1 \frac{\mathrm{d}H_1}{\mathrm{d}t} = Q_1 - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2}, \tag{5a}$$

$$A_2 \frac{\mathrm{d}H_2}{\mathrm{d}t} = Q_2 - \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2}, \tag{5b}$$

where A_1, A_2 are the cross-sectional area of Tank #1, #2; H_1, H_2 are the liquid level in Tank #1, #2; Q_1, Q_2 are the volumetric flow rate (cm^3/s) of Pump #1, #2; α_1, α_2, α_3 are the proportionality constant corresponding to the $\sqrt{H_1}$, $\sqrt{H_2}$ and $\sqrt{H_1 - H_2}$ terms. Note that here we assume $H_1 \geqslant H_2$, which is always satisfied in the experiments.

The coupled-tank apparatus can be configured as a second-order single-input single-output system by turning off Pump #2 and using Pump #1 to control the water level in Tank #2. Since Pump #2 is turned off, Q_2 equals zero and Eq. (5b) reduces to

$$A_2\frac{\mathrm{d}H_2}{\mathrm{d}t} = -\alpha_2\sqrt{H_2} + \alpha_3\sqrt{H_1 - H_2}. \tag{6}$$

Eqs. (5a) and (6) are used to construct a simulation model of the coupled tank for the GA to evaluate the fitness of the candidate solutions. The parameters used in the simulation model are as follows:

$A_1 = A_2 = 36.52\,\text{cm}^2$,

$\alpha_1 = \alpha_2 = 5.6186$,

$\alpha_3 = 10$.

The area of the tank was measured manually while the discharge coefficients (α_1, α_2 and α_3) were found by measuring the time taken for a pre-determined change in the water levels to occur. Although the DC power source can supply between 0 and 5 V to the pumps, the maximum control signal is capped at 4.906 V which corresponds to an input flow rate of about $75\,\text{cm}^3/\text{s}$. To compensate for the pump dead zone, the minimum control signal is chosen to be 1.646 V. A sampling period of 1 s is employed.

5.2. GA parameters

The model of the coupled-tank apparatus described in the previous subsection is constructed using physical laws and does not accurately reflect the characteristics of the practical plant. For example, it has been documented that the volumetric flow rate of the pumps in the coupled-tank apparatus used to produce the results is non-linear, the system has non-zero transport delay and the sensor output is noisy (Teo et al., 1998). Due to the presence of such modelling uncertainties, the performance of the FLCs designed using the simulation model will inevitably deteriorate when they are applied to the real-world problem. This work aims at studying whether the FOU of the type-2 FLC will enable it to cope better with the modelling uncertainties. To find the best possible FOU, there is a need to expose the FLCs to uncertain model parameters during the design phase because the input–output mapping of the type-2 FLC is fixed once the controller parameters are selected. Hence, 4 plants (I–IV) with the parameters shown in Table 3 are used to evaluate each chromosome. The sum of the integral of the time-weighted absolute errors (ITAEs) obtained from the 4 plants, defined as Eq. (7), is used by the GA to evaluate the fitness of each candidate solution. It is taken to be the

$$F = \sum_{i=1}^{4}\alpha_i\left[\sum_{j=1}^{N_i} j \times |e_i(j)|\right], \tag{7}$$

where $e_i(j)$ is the difference between the setpoint and the actual liquid height at the jth sampling of the ith plant, α_i is the weight corresponding to the ITAE of the ith plant, and $N_i = 200$ is the number of sampling instants. There is a need to introduce α_i because the ITAE of the second plant is usually several times bigger than that of other plants. To ensure that the ITAE of the four plants can be reduced with equal emphasis, α_2 is defined as $\frac{1}{3}$ while the other weights are unity.

The GA parameters used to evolve the MFs of all the four FLCs in this paper are the same. A population size of 100 chromosomes coded in real number is used. Members of the first generation are randomly initialized and the GA terminates after 600 generations. The termination point was selected after an inspection of the fitness function verses generation plot revealing that the fitness function will settle within 600 generations. To ensure that the fitness function decreases monotonically, the best candidate solution in each generation enters the next generation directly. In addition, a generation gap of 0.8 is used during the reproduction operation so that 80% of the members in the new generation are determined by the selection scheme employed, while the remaining 20% are selected randomly from the pre-defined search domain. This strategy helps to prevent premature convergence of the population. The crossover rate is 0.8 and the mutation rate is 0.1. One-point crossover operator is employed. In order to enable finer adjustment to occur as the generation number (i) becomes bigger, the non-linear mutation (Sakawa, 2002) method defined in Eq. (8) is used in the FLC design

$$x(i+1) = x(i) + \delta(i), \tag{8}$$

where

$$\delta(i) = \begin{cases} a \cdot [1 - \lambda^{(1-(i/i_{\max}+1))}] & \text{if } \text{rand}(1) > 0.5, \\ -a \cdot [1 - \lambda^{(1-(i/i_{\max}+1))}] & \text{otherwise.} \end{cases}$$

$x(i)$ is the value of gene x in ith generation, $i_{\max}$ is the maximum number of generations, λ and rand(1) are random numbers in $[0, 1]$, and a is a constant associated with each input and output. In this paper a for each input is chosen to be $\frac{1}{6}$ of the length of its universe of discourse, and a for the output is $\frac{1}{10}$ of the length of its universe of discourse. Flexible position-coding strategy is applied in each input or output domain to improve the diversity of the members in each generation. Consequently, the genes in each sub-chromosome may not remain in the proper order after crossover and mutation, i.e. the center of the type-2

Table 3
Plants used to assess fitness of candidate solutions

	I	II	III	IV
$A_1 = A_2$ (cm^2)	36.52	36.52	36.52	36.52
$\alpha_1 = \alpha_2$	5.6186	5.6186	5.6186	5.6186
α_3	10	10	10	8
Setpoint (cm)	$0 \to 15$	$0 \to 22.5 \to 7.5$	$0 \to 15$	$0 \to 15$
Transport delay (s)	0	0	2	0

set corresponding to N_e may be larger than that of Z_e. Every sub-chromosome is, therefore, sorted before fitness evaluation is performed.

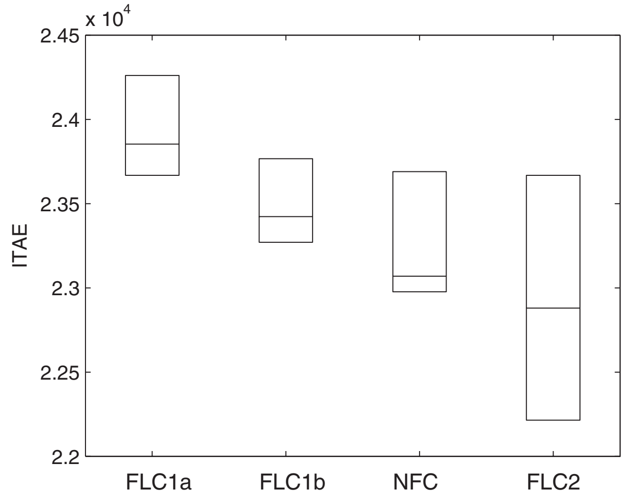

Fig. 7. Statistical properties of the 4 FLCs.

As GA is a stochastic method, a statistical evaluation is performed. The optimization routine for each of the 4 FLCs were repeated 10 times and the ITAEs of the best chromosomes were recorded. The results are illustrated using the box plot in Fig. 7, where the lower limit of each box is the best performance, the upper limit is the worst performance and the line in between is the mean performance. It may be observed that, on average, FLC_2 outperforms the other 3 kinds of FLCs. The mean ITAE of the FLC_2 is also lower than the best ITAEs of the other 3 controllers. Another interesting observation is that FLC_2 has the biggest variance among the four, though it does not have the largest number of parameters. This characteristic may be explained using the concept of equivalent type-1 sets (ET1Ss) (Wu and Tan, 2005a), which interprets the FOU as a group of equivalent type-1 fuzzy sets (ET1Ss). When one parameter of a type-2 FLC is changed, an entire collection of the ET1Ss will vary. In contrast, only one MF is affected when one parameter of a type-1 FLC is changed. As the influence of parameter change in a type-2 FLC is much bigger than that in a type-1 FLC, the variance of type-2 FLCs may be bigger.

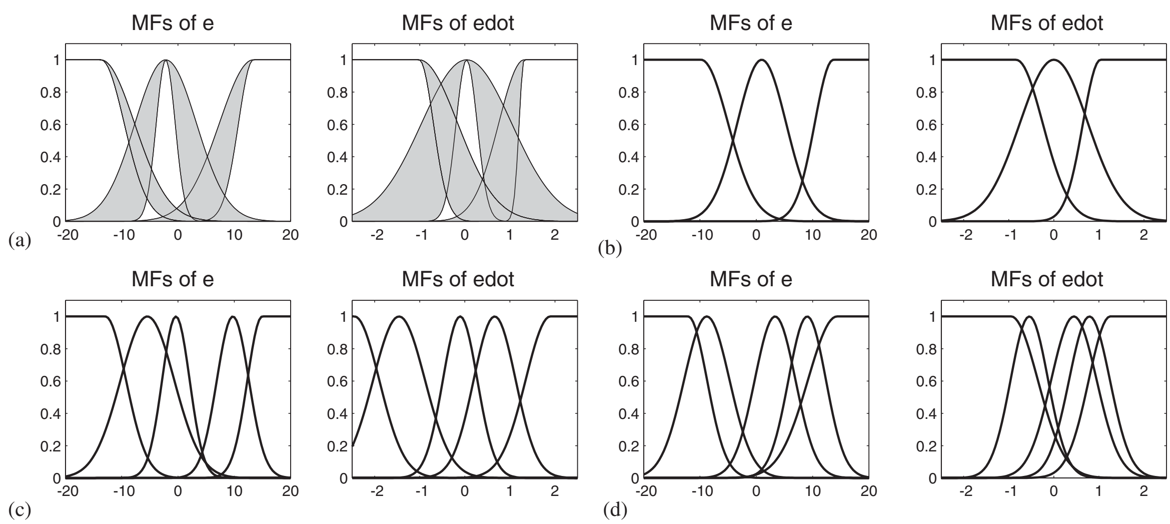

Fig. 8. MFs of the four FLCs. 'edot' represents $\dot{e}$ in all the figures: (a) MFs of FLC_2; (b) MFs of FLC_{1a}; (c) MFs of FLC_{1b}; (d) MFs of NFC.

Table 4
MFs of the type-2 FLC FLC_2

Input		*N*	*Z*	*P*
(a) *MFs of the inputs*				
e	m	−13.6778	−2.1764	13.3864
	$[\delta_1, \delta_2]$	[4.1385, 5.9727]	[1.6850, 5.4645]	[2.6457, 6.0475]
$\dot{e}$	m	−1.0132	0.0393	1.3172
	$[\delta_1, \delta_2]$	[0.3172, 0.8553]	[0.2342, 1.0000]	[0.1130, 0.5656]
(b) *MFs of the output*				
$\dot{u}_1$	$\dot{u}_2$	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_5$
−0.8091	−0.3429	0.0796	0.4656	0.7457

Table 5
MFs of the type-1 FLC FLC_{1a}

	N_e	Z_e	P_e	$N_{\dot{e}}$	$Z_{\dot{e}}$	$P_{\dot{e}}$
(a) *MFs of the inputs*						
m	−9.7890	0.9611	13.6741	−0.8344	0.0022	1.0366
δ	4.7869	4.3414	3.3040	0.5887	0.7562	0.3902
(b) *MFs of the output*						
$\dot{u}_1$	$\dot{u}_2$	$\dot{u}_3$	$\dot{u}_4$	$\dot{u}_5$		
−0.3449	−0.1406	0.0668	0.6201	0.8899		

Table 6
MFs of the type-1 FLC FLC_{1b}

Input		NB	NM	Z	PM	PB
(a) *MFs of the inputs*						
e	m	−12.9009	−5.4265	−0.3698	9.7432	14.9622
	δ	3.6845	4.7648	2.4434	2.8409	2.6371
$\dot{e}$	m	−2.4483	−1.4590	−0.1044	0.6618	1.8987
	δ	0.5499	0.5756	0.3823	0.4662	0.5499

e	$\dot{e}$				
	$NB_{\dot{e}}$	$NM_{\dot{e}}$	$Z_{\dot{e}}$	$PM_{\dot{e}}$	$PB_{\dot{e}}$
(b) *Rule base and consequents*					
NB_e	−0.7999	−0.6734	−0.2558	−0.1375	−0.0096
NM_e	−0.6734	−0.2558	−0.1375	−0.0096	0.2468
Z_e	−0.2558	−0.1375	−0.0096	0.2468	0.5219
PM_e	−0.1375	−0.0096	0.2468	0.5219	0.7295
PB_e	−0.0096	0.2468	0.5219	0.7295	0.8595

Table 7
MFs of the neuro-fuzzy controller, NFC

Input		NB	NM	Z	PM	PB
(a) *MFs of the inputs*						
e	m	−12.0948	−8.7795	3.3386	9.0337	14.3214
	δ	3.3558	4.0363	3.6185	3.3313	5.1559
$\dot{e}$	m	−0.9471	−0.5429	0.4458	0.7916	1.2536
	δ	0.5923	0.4165	0.5046	0.4531	0.4781

e	$\dot{e}$				
	$NB_{\dot{e}}$	$NM_{\dot{e}}$	$Z_{\dot{e}}$	$PM_{\dot{e}}$	$PB_{\dot{e}}$
(b) *Rule base and consequents*					
NB_e	−0.3052	−0.3671	−0.3679	−0.1765	−0.3744
NM_e	−0.0074	0.2593	−0.1646	0.1657	−0.2463
Z_e	−0.0998	−0.0742	−0.0590	0.2755	0.2225
PM_e	−0.2286	0.1267	0.3988	0.1868	0.2677
PB_e	0.6778	0.1248	0.4227	0.1317	0.0201

Fig. 8 shows the MFs of the four FLCs that have the lowest ITAE. The parameters of the FLCs are listed in Tables 4–7. The fitness value verses generation number curves of the four GAs are shown in Fig. 9. It indicates that the fitness values have converged. Another observation is that the additional mathematical dimension provided by the FOU enables FLC_2 to achieve a lower ITAE than the other three type-1 FLCs, though FLC_2 has less parameters

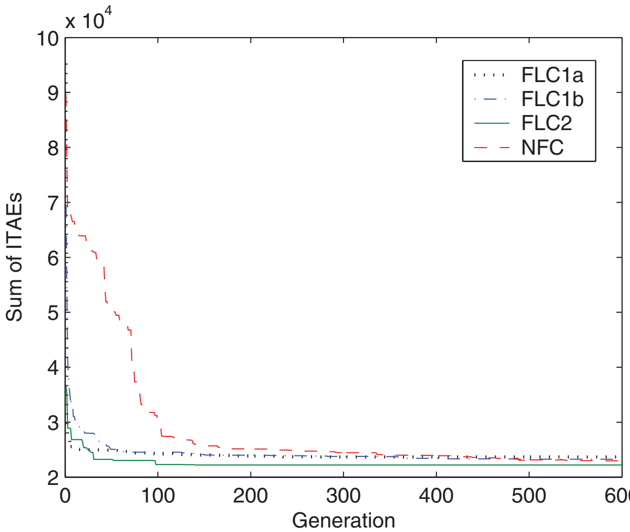

Fig. 9. Relationship between generation and sum of ITAE.

than two of the type-1 FLCs. To further assess the performance of the FLCs, simulation and experimental study was conducted and the results are presented in the following subsection.

5.3. Performance study

Fig. 10 shows the step responses and the corresponding control signals obtained when the four FLCs were used to control the nominal plant. Performances of FLC_2, FLC_{1a} and FLC_{1b} are comparable to NFC, a neurofuzzy controller reported in the literature (Teo et al., 1998). The results also indicate that the FLCs evolved by GA are able to provide satisfactory control in spite of the pump non-linearity and the unmodelled transport delay.

To test the ability of the FLCs to handle unmodelled dynamics, transport delay was deliberately introduced into the feedback loop. First, a transport delay equal to 1 s (one sampling period) was artificially added to the nominal system. The step responses and the control signal are shown in Fig. 11. When a 2 sampling periods transport delay was added to the system, the corresponding step responses and the control signal are shown in Fig. 12. Although the simulation results indicated that the four FLCs should have similar performances, large oscillations were obtained when FLC_{1a} and FLC_{1b} were used to control the actual plant. FLC_2 and NFC produced experimental results that match the simulation results more closely. Of the four controllers, FLC_2 provided the best experimental performance as its step response has the smallest overshoot and is least oscillatory.

Next, the ability of the FLCs to cope with variations in the system dynamics was investigated by lowering the baffle separating the two tanks so that the discharge coefficient between the two tanks (α_3) was reduced from 10 to 8. Since the simulation model indicates that the steady-

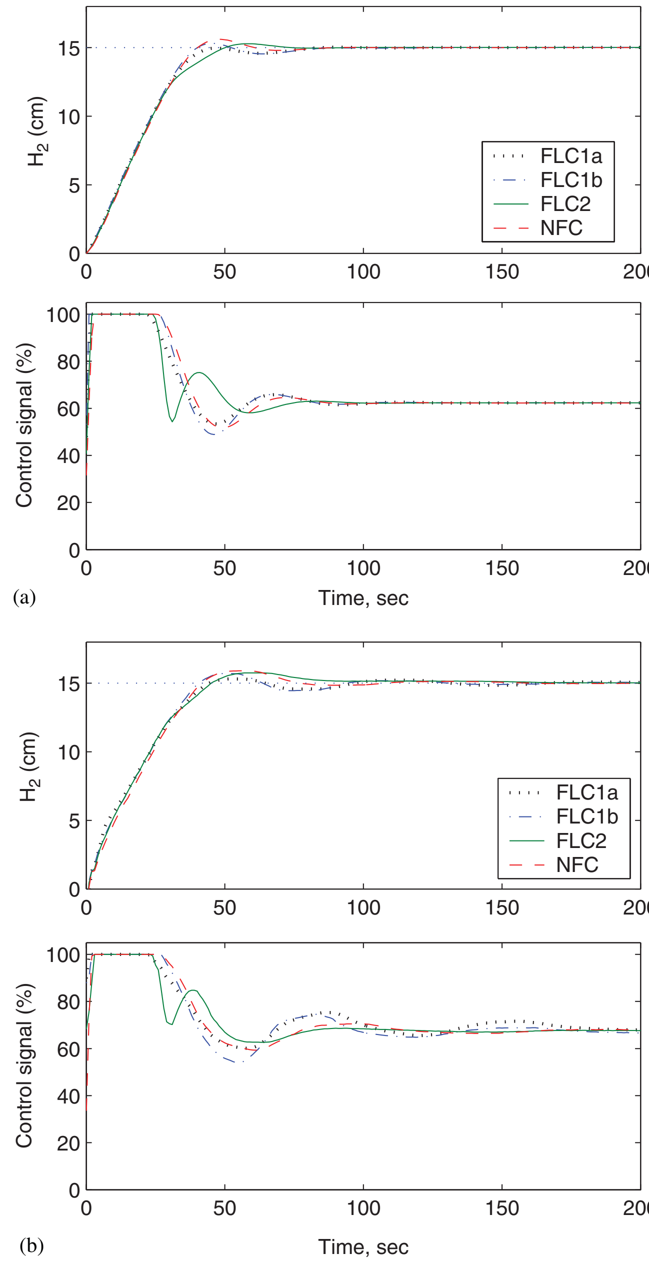

Fig. 10. Step responses for the nominal plant: (a) Simulation results; (b) experiment results.

state water level in tank #1 and tank #2 is 22.4 and 15 cm, respectively, when $\alpha_3 = 8$, the baffle was lowered until the liquid level in the two tanks are at the above-mentioned heights. The resulting system is more sluggish and the steady-state difference in liquid level between the two tanks was larger. The step responses and the control signal are shown in Fig. 13. Fig. 14 shows the step responses and the control signal when a 1-s transport delay was added to the modified plant. From the step responses, it may be observed that FLC_{1a} gave the poorest control performance. Though the liquid level in the tank eventually reached the desired setpoint, the settling time was so long

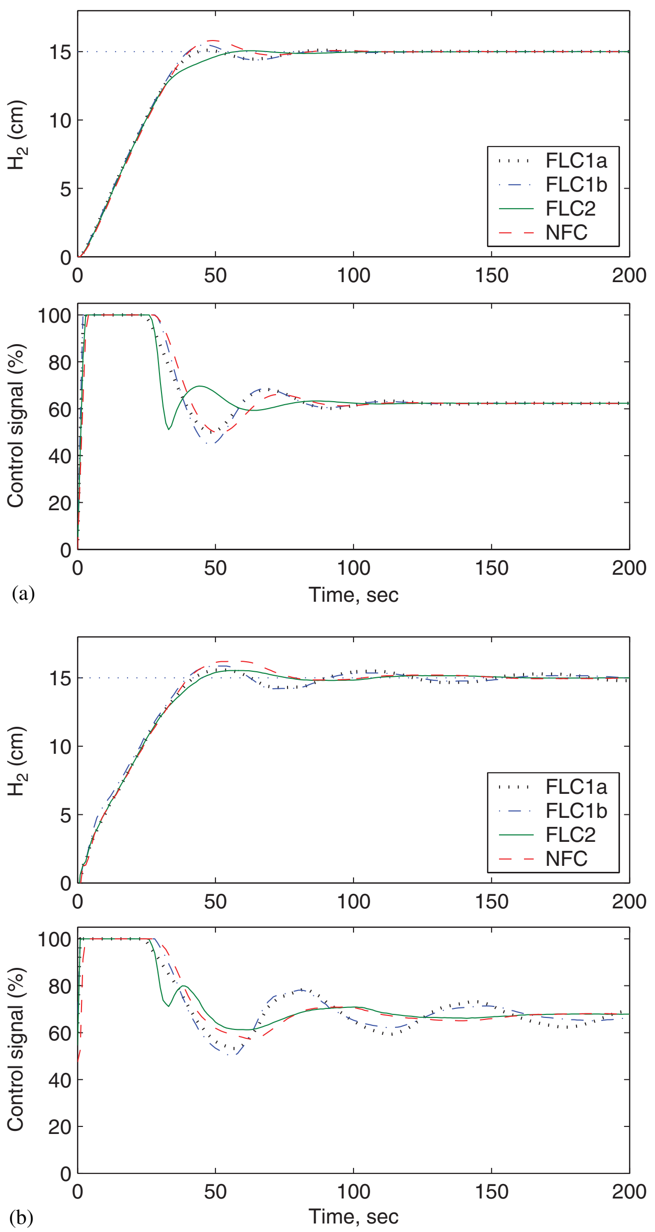

Fig. 11. Step responses when a 1 s transport delay was added to the nominal plant: (a) Simulation results; (b) experimental results.

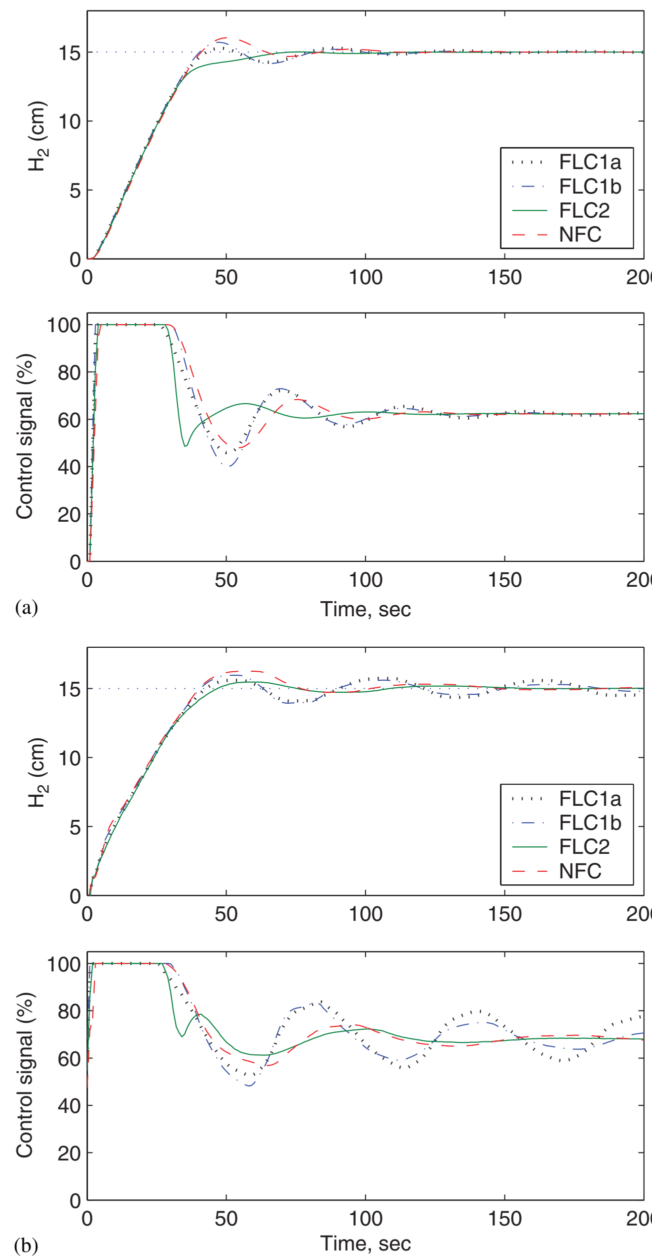

Fig. 12. Step responses when a 2 s transport delay was added to the nominal plant: (a) Simulation results; (b) experimental results.

that it was inconvenient to plot the complete trajectory in the figures. The least oscillatory response was provided by FLC_2.

6. Discussions

A consolidation of the simulation and experimental results obtained during the comparative study are presented in Table 8. Integral of the Time Absolute Error (ITAE) is used as the performance index. The deterioration in performances when the test platform is switched from simulation to the physical plant reflect the abilities of the four FLCs to handle modelling uncertainties. Another finding is that FLC_2 outperforms the other 3 controllers, even though NFC and FLC_{1b} has, respectively, $45-23=22$ and $29-23=6$ more parameters (degree of freedom) than FLC_2. The study suggests that a type-2 FLC can provide better performance with less MFs and a smaller rule base, making it more appealing than its type-1 counterpart with regards to accuracy and interpretability.

In order to gain some insights into why the type-2 FLC is able to achieve better control performances, the control surface of the four controllers were plotted. Fig. 15 shows that the control surface of the type-2 FLC is more complex. It may be observed that the control surface of the type-2 FLC has a gentler gradient around the equilibrium point

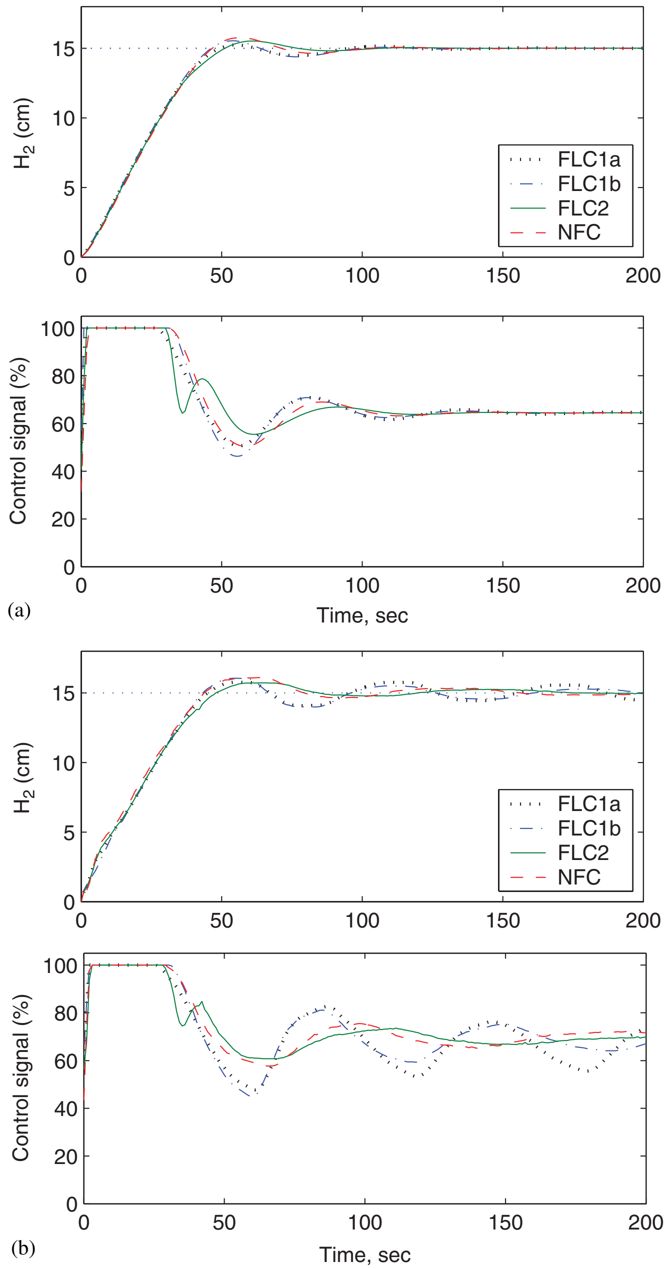

Fig. 13. Step responses when the baffle was lowered: (a) Simulation results; (b) experimental results.

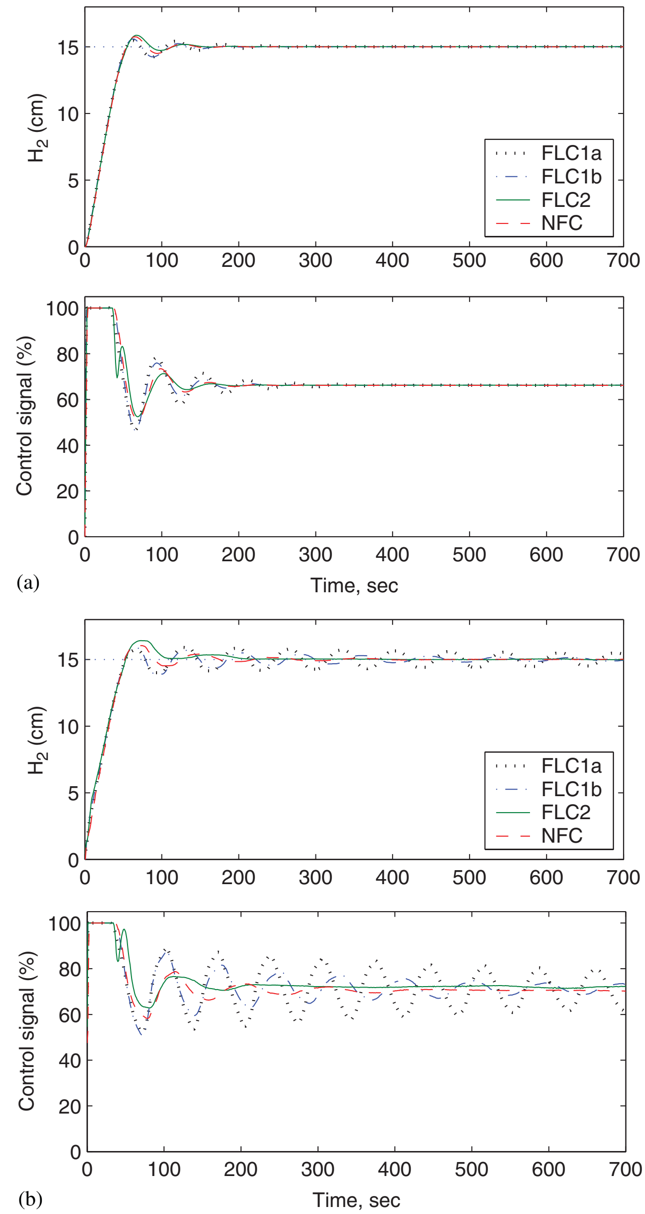

Fig. 14. Step responses when the baffle was lowered and a 1 s transport delay was added: (a) Simulation results; (b) experimental results.

$(e = 0,\ \dot{e} = 0)$. As a result, the changes in the output control signal are small in this area and small disturbances around the equilibrium point will not result in significant control signal change. This behaviour may help to explain why the type-2 FLC is better able to attenuate oscillations. To illustrate the idea more clearly, a slice of the control surface at $\dot{e} = 0$ is shown in Fig. 16. It is observed that the outputs of the four controllers are similar when $e \in [0, 0.5]$. However, when $e < 0$, FLC_2 has a gentler slope so the absolute values of $\dot{u}$ is smaller compared to those of FLC_{1a}, FLC_{1b} and NFC. The implication is that an overshoot will decay away more gradually, and thus reducing the amount of oscillations. This conclusion is consistent with the results in Figs. 10–14, where there are much fewer oscillations when FLC_2 is employed.

Besides performance, the computational cost required to implement the controllers is also an important consideration. The GAs used to tune the four FLCs were implemented as a Matlab 6.5 program and executed on an Intel Pentium III 996 MHz computer with 256M RAM. The time needed by the four GAs to complete 100 generations of evolution was recorded and shown in Table 8. A 10,000 time-step simulation (the setpoint is $15 + 10\sin(i/50)$, where $i = 1, 2, \ldots, 10{,}000$ is the time instant) using the evolved FLCs was also carried out on the same computer and the computation time is presented

Table 8
A comparison of the four FLCs

Structure	Type		FLC_{1a}	FLC_{1b}	NFC	FLC_2
			Type-1	Type-1	Type-1	Type-2
Performance	No. input MFs		3	5	5	3
(ITAE)	No. output MFs		5	9	25	5
	Total parameters		17	29	45	23
	Plant I	Simulation	4487	4501	4491	4577
		Experimental	6236	6177	6082	6516
	Plant I	Simulation	4970	4995	5030	4907
	1s delay	Experimental	8784	7755	6957	5665
	Plant III	Simulation	6165	6033	5909	5482
		Experimental	12394	10051	7543	5927
	Plant IV	Simulation	6611	6583	6673	6284
		Experimental	13114	11038	8813	7807
	Plant IV	Simulation	9176	9458	7756	8334
	1s delay	Experimental	109820	49584	17731	17511
Computation	GA tuning (s)		950	1050	1300	5860
time	Simulation (s)		1.4070	1.5780	1.8120	8.1720

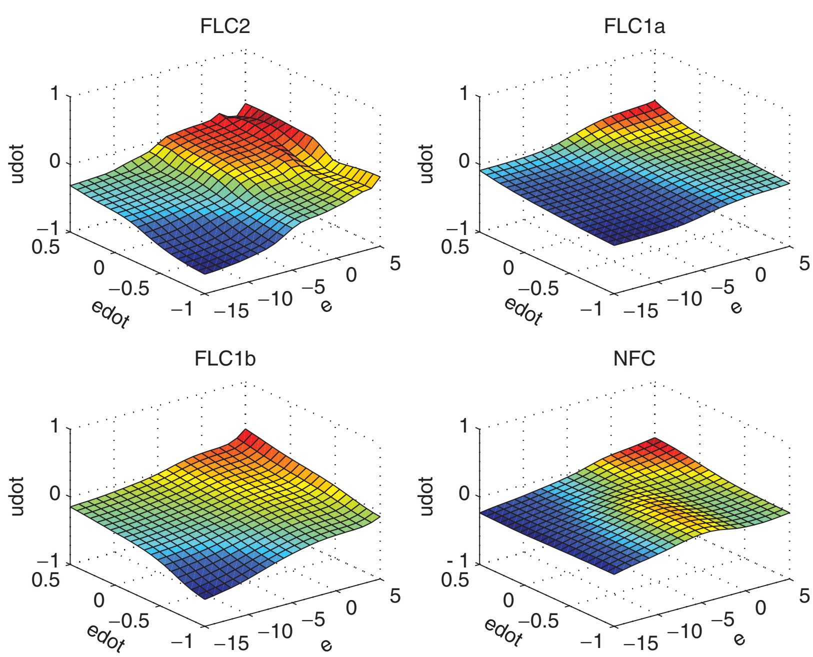

Fig. 15. Control surface of the four FLCs. 'edot' represents $\dot{e}$ and 'udot' represents $\dot{u}$.

in Table 8. The data indicate that the computational cost of FLC_2 is higher than that of the other 3 controllers. The increase in computational burden is mainly due to the type-reducer. While the need for large computing power is a hindrance to real-time implementation, methods to reduce the computational requirements of type-2 FLS are available (Wu and Mendel, 2002; Wu and Tan, 2005b).

7. Conclusions

In this paper, a GA-based totally independent method is used to design a type-2 FLC for controlling a coupled-tank liquid-level control system. The performance of the type-2 FLC (23 parameters) is compared with that of three type-1 FLCs: a type-1 Mamdani FLC with 17 parameters, a type-1 Mamdani FLC with 29 parameters and a type-1 neuro-fuzzy controller with 45 parameters. The results demonstrate that a type-2 FLC can outperform type-1 FLCs that have more design parameters. Thus, the type-2 FLC is more appealing than its type-1 counterpart with regards to accuracy and interpretability. The main advantage of the type-2 FLC appears to be its ability to eliminate persistent oscillations, especially when unmodelled dynamics were introduced. This ability to handle modelling error is

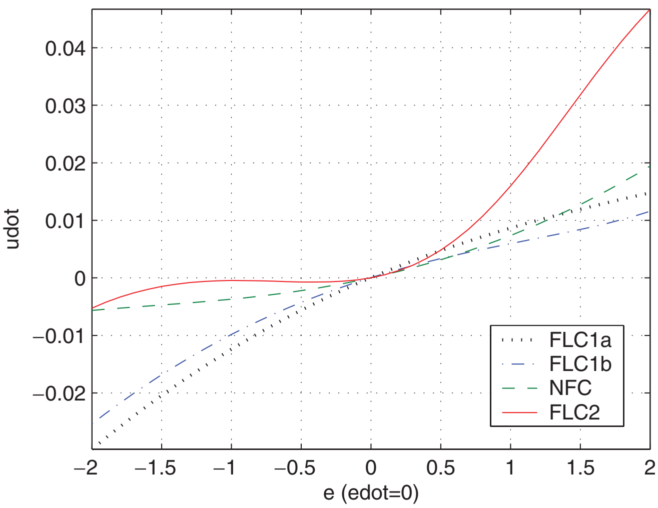

Fig. 16. A slice of the control surfaces at $\dot{e} = 0$.

particularly useful when FLCs are tuned offline using GA and a model as the impact of unmodelled dynamics is reduced.

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