A Type-2 Fuzzy Logic Controller for the Liquid-level Process

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Abstract—**This paper focuses on evolving type-2 fuzzy logic controllers (FLCs) genetically and examining whether they are better able to handle modelling uncertainties. The study is conducted by utilizing a type-2 FLC, evolved by a genetic algorithm (GA), to control a liquid-level process. A two stage strategy is employed to design the type-2 FLC. First, the parameters of a type-1 FLC are optimized using GA. Next, the footprint of uncertainty is evolved by blurring the fuzzy input set. Experimental results show that the type-2 FLC copes well with the complexity of the plant, and can handle the modelling uncertainty better than its type-1 counterpart.**

I. Introduction

Fuzzy logic is a form of logic whose underlying modes of reasoning are approximate instead of exact. Unlike crisp logic, it emulates the ability to reason and use approximate data to find solutions. Fuzzy logic controllers (FLCs) are knowledge-based controllers consisting of linguistic "IF-THEN" rules that can be constructed using the knowledge of experts in the given field of interest. FLCs have demonstrated their ability in a number of applications [1], especially for the control of complex nonlinear systems that may be difficult to model analytically [2], [3].

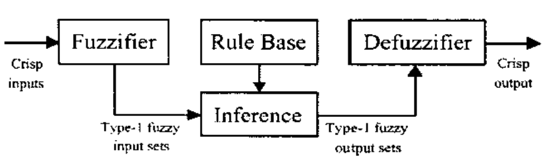

Fig. 1. A type-1 fuzzy logic system

As illustrated in Fig. 1, a traditional fuzzy logic system consists of 4 components— the rule base, the fuzzy inference engine, the fuzzifier and the defuzzifier. Knowledge is embedded within the rule base in the form of rules whose antecedent and consequent are fuzzy sets that partition the input and output domains. Despite having a name that has the connotation of uncertainty, researches have shown that type-1 fuzzy logic systems have difficulties in modelling and minimizing the effect of uncertainties [4]. The main reason is that a type-1 fuzzy set is certain in the sense that for each input, there is a crisp membership grade. Recently, a type of fuzzy sets characterized by membership grades that are themselves fuzzy have been attracting interest [4]. As illustrated in Fig. 2, a type-2 fuzzy MF can be obtained by starting with a type-1 MF and blurring it. The extra mathematical dimension provided by the blurred area, referred to as the footprint of uncertainty (FOU), represents the uncertainties in the shape and position of the type-1 fuzzy set. The FOU is bounded by upper and lower MFs, and points within the "blurred area" have membership grades given by type-1 MFs. The most frequently used type-2 fuzzy sets are interval fuzzy sets, where each point in the FOU has unity secondary membership grade.

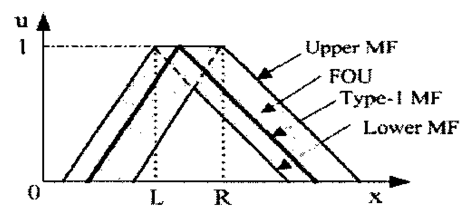

Fig. 2. A type-2 fuzzy set

A fuzzy logic system described using at least one type-2 fuzzy set is called a type-2 fuzzy logic system. They are very useful in circumstances where it is difficult to determine an exact membership grade for a fuzzy set; hence, they can be used to handle system uncertainties and have the potential to outperform their type-1 counterparts. Fig. 3 shows the structure of a type-2 fuzzy logic system. It is similar to its type-1 counterpart, the major difference being that at least one of the fuzzy sets is type-2 and a type-reducer is needed to convert the type-2 fuzzy output sets into type-1 sets so that they can be processed by the defuzzifier to give a crisp output.

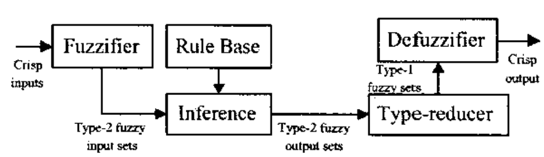

Fig. 3. A type-2 fuzzy logic system

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Perhaps the most difficult problem in FLC design is to determine the MFs and the rule base. Genetic algorithm (GA) is widely used recently to solve this problem [5], [6], [7]. GA is a general-purpose search algorithm that uses principles inspired by natural population genetics to evolve solutions to problems. It was first proposed by Holland in 1975. GAs are theoretically and empirically proven to provide a robust search in complex spaces, thereby offering a valid approach to problems requiring efficient and effective searches [8], [9], [10].

GA performs optimization by processing a population of generic variants from one generation to the next. A particular chromosome encodes a candidate solution of the optimization problem. The fitness of an individual with respect to the optimization task is described by a scalar objective function (fitness function). According to Darwin's principle, highly fit individuals are more likely to be selected to reproduce offsprings. Genetic operators such as crossover and mutation are applied to the parents in order to generate new candidate solutions. As a result of this evolutionary cycle of selection, crossover and mutation, more and more suitable solutions to the optimization problem emerge within the population.

This paper aims at developing a strategy for using GA to design a type-2 FLC. A comparison of the abilities of type-1 and type-2 FLCs to handle modelling uncertainties is also performed. The rest of the paper is organized as follows: Section II describes a singleton type-2 FLC. Next, the coupled-tank liquid-level system is introduced. Two FLCs, one type-1 and one type-2, are designed in Section IV and their abilities to handle modelling uncertainties are experimentally compared in Section V. Section VI discusses issues on computational cost. Finally, conclusions are drawn in Section VII.

II. SINGLETON INTERVAL TYPE-2 FLC

This section discusses the singleton interval type-2 FLC that is studied hereafter. As a singleton interval type-2 FLC fuzzifies crisp input signals as singletons, it does not account explicitly for input measurement uncertainties. The antecedents of the rule base are constructed using interval type-2 fuzzy sets in order to handle model uncertainties, while the consequents are type-1 fuzzy sets.

A. Fuzzy Inference Engine

The role of the fuzzy inference engine is to combine the rules which are fired in order to generate a mapping from crisp inputs to output type-2 fuzzy sets. Just as the sup-star composition is the backbone computation for a type-1 FLC, the extended sup-star composition is the backbone for a type-2 FLC. Each rule is interpreted as a type-2 fuzzy implication. Suppose the sum-min inference engine is used and a rule is

$$R^l : \widetilde{F}_1^l \ and \ \widetilde{F}_2^l \rightarrow G^l$$

$\widetilde{F}_1^l$ and $\widetilde{F}_2^l$ are interval type-2 fuzzy sets while G^l is a type-1 fuzzy set. As all the type-2 sets used here are interval ones, the result of the input and antecedent operations, involved in calculating the firing set $\sqcap_{i=1}^{p}\mu_{\widetilde{F}_i^l}(x_i) \equiv F^l(\mathbf{x})$, is an interval type-1 set :

$$F^l(\mathbf{x}) = [\underline{f}^l(\mathbf{x}), \overline{f}^l(\mathbf{x})] \equiv [\underline{f}^l, \overline{f}^l] \tag{1}$$

where $\underline{f}^l(\mathbf{x}) = min\left(\underline{\mu}_{\widetilde{F}_1^l}(x_1),\ \underline{\mu}_{\widetilde{F}_2^l}(x_2)\right)$
$\overline{f}^l(\mathbf{x}) = min\left(\overline{\mu}_{\widetilde{F}_1^l}(x_1),\ \overline{\mu}_{\widetilde{F}_2^l}(x_2)\right)$
$\underline{\mu}_{\widetilde{F}^l}(x_i)$ is the lower membership grade of $\widetilde{F}^l$
$\overline{\mu}_{\widetilde{F}^l}(x_i)$ is the upper membership grade of $\widetilde{F}^l$.

The output set that is obtained when rule R^l is fired is the following type-2 fuzzy set

$$\mu_{\widetilde{B}^l}(y) = \int_{b^l \in \left[\underline{f}^l(\mathbf{X})\star\mu_{G^l}(y), \overline{f}^l(\mathbf{X})\star\mu_{G^l}(y)\right]} 1/b^l, \quad y \in Y \tag{2}$$

where $\mu_{G^l}(y)$ is the membership grade of $G^l(y)$. To determine $\mu_{\widetilde{B}^l}(y)$, one only needs to compute its lower and upper membership grades.

B. Type-Reduction

The outputs corresponding to the fired rules are type-2 fuzzy sets which must be type-reduced before the defuzzifier can be used to generate a crisp output. This is the main structural difference between the type-1 and type-2 FLCs. In this paper, the center-of-sets type-reducer is used. It combines all the type-2 output sets and then performs a center-of-sets calculation to produce a type-1 set, known as a type-reduced set. There are two type-reduction methods: Karnik-Mendel iterative method [4] and the uncertainty bound method [11]. The first approach is adopted and it is based on the Generalized Centroid (GC) concept [4] which may be defined as

$$\begin{aligned} GC &\\ &= \int_{z_1\in Z_1}\cdots\int_{z_N\in Z_N}\int_{w_1\in W_1}\cdots\int_{w_N\in W_N} 1\Bigg/\frac{\sum_{i=1}^{N} z_i w_i}{\sum_{i=1}^{N} w_i} \\ &= [y_l,\ y_r] \end{aligned} \tag{3}$$

where Z_i is a type-1 set with center c_i and spread s_i, and W_i is a type-1 set with center h_i and spread δ_i. z_i and w_i are calculated using c_i, s_i, h_i and δ_i via the following iterative procedure shown in Fig. 4. It has been proved that this iterative procedure can converge in at most N iterations [4].

C. Defuzzification

Once y_l and y_r are obtained, they can be used to calculate the crisp output. Since the type-reduced set is an interval set, the output is $(y_l + y_r)/2$.

III. THE COUPLED-TANK SYSTEM

The plant used as the test bed in this work is the coupled-tank apparatus shown in Fig. 5. The equipment consists of two small tower-type tanks mounted above a reservoir that store the water. Water is pumped into the top of each tank by two independent pumps, and the levels of water are measured by

Set $z_i = c_i + s_i$ *for* $i = 1, \ldots, N$;
Arrange z_i *in ascending order;*
Set $w_i = h_i$ *for* $i = 1, \ldots, N$;
$y' = \frac{\sum_{i=1}^{N} z_i w_i}{\sum_{i=1}^{N} w_i}$;
$y'' = y'$;
do
  $y' = y''$;
  Find $k \in [1, N-1]$ *such that* $z_k \leq y' \leq z_{k+1}$;
  Set $w_i = h_i - \delta_i$ *for* $i \leq k$
  Set $w_i = h_i + \delta_i$ *for* $i \geq k+1$;
  $y'' = \frac{\sum_{i=1}^{N} z_i w_i}{\sum_{i=1}^{N} w_i}$;
while $y' != y''$
$y_r = y''$;

Fig. 4. Karnik-Mendel iterative method

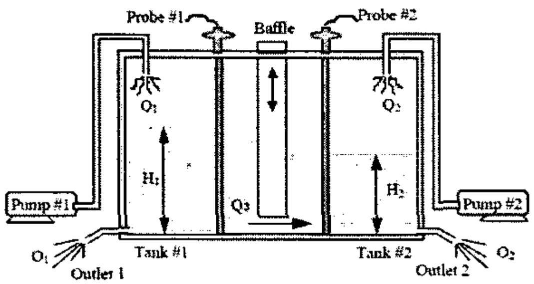


Fig. 5. The coupled-tank liquid-level control system

two capacitive-type probe sensors. Each tank is fitted with an outlet, at the side near the base. Raising the baffle between the two tanks allows water to flow between them. The amount of water which returns to the reservoir is approximately proportional to the square root of the height of water in the tank, which is the main source of nonlinearity.

The dynamics of the coupled-tank apparatus can be modelled by the following set of nonlinear differential equations:

$$A_1 \frac{dH_1}{dt} = Q_1 - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2} \quad \text{(4a)}$$

$$A_2 \frac{dH_2}{dt} = Q_2 - \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \quad \text{(4b)}$$

where A_1, A_2 are the cross-sectional area of Tank #1, #2; H_1, H_2 are the liquid level in Tank #1, #2; Q_1, Q_2 are the volumetric flow rate (cm^3/sec) of Pump #1, #2; α_1, α_2, α_3 are the proportionality constant corresponding to the $\sqrt{H_1}$, $\sqrt{H_2}$ and $\sqrt{H_1 - H_2}$ terms.

The coupled-tank apparatus is configured as a second-order single input single output (SISO) system in this paper by turning off Pump #2 and using Pump #1 to control the water level in Tank #2. In this case, Q_2 equals zero and Equation (4b) can be simplified to :

$$A_2 \frac{dH_2}{dt} = -\alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \quad (5)$$

FLCs were tuned based on the simulated plant. The sampling period is 1 sec. For the results reported herein, the following parameters are adopted:

$$A_1 = A_2 = 36.52 \text{ cm}^2$$
$$\alpha_1 = \alpha_2 = 5.6186$$
$$\alpha_3 = 10$$

The maximum control signal is 5 volts, corresponding to an approximately flow rate of pump #1 of 73 cm^3/sec. These parameters were identified using data collected from the physical plant so that the FLCs designed using the simulated process can be tested experimentally.

IV. Design of FLCs

In this section, a GA based strategy for designing a singleton interval type-2 FLC is proposed.

Two very different approaches may be used to select the parameters of a type-2 fuzzy logic system [4]. One is a partially dependent approach, where a best possible type-1 fuzzy logic system is designed first, and then used to initialize the parameters of a singleton type-2 fuzzy logic system. The other method is a totally independent approach, where all of the parameters of the type-2 FLC are tuned from scratch without the aid of an existing type-1 design. In this paper, the partially dependent approach is adopted because it has the following benefits: 1) smart initialization of the parameters of the type-2 FLC; and 2) a baseline design whose performance can be compared with that of the type-2 FLC [4]. Another benefit is the number of parameters that need to be tuned is usually fewer with this approach, thus the GA can converge at a faster speed.

The partially dependent approach requires a good baseline design. To satisfy this requirement, the proposed design strategy is to use GA to optimize the parameters of a type-1 FLC. The type-2 FLC is then evolved by blurring the type-1 fuzzy input sets to generate the FOU. Before presenting details about how GA is used to tune the FLCs, the structure of the type-1 and type-2 FLC that is used to control the liquid level process is described.

In order to concentrate on examining the possibility of using GA to evolve type-2 FLCs and to assess their ability to handle uncertainties, the standard fuzzy Proportional plus Integral (PI) controller is employed. Hence, the input signals of the FLCs are the feedback error, e, and the change of the error, $\dot{e}$, while the output signal is the change in the control signal, $\dot{u}$. Each input domain is partitioned by three fuzzy MFs that are labelled as **N**, **Z** and **P**. The output space has five MFs labelled as **NB**, **NS**, **Z**, **PS** and **PB**. Table I shows the fuzzy rule base, which is commonly used to construct fuzzy logic controllers. Both the type-1 and type-2 FLCs have essentially the same architecture. The only difference being that the input domains of the type-1 and type-2 FLC are partitioned by type-1 and interval type-2 fuzzy sets respectively. As described in the previous section, the center-of-sets type-reducer and the height defuzzifier are used to calculate the crisp control signal.

A. The Genetic Algorithms

In this paper, the MFs of the type-1 FLC and the FOU of the type-2 FLC are evolved using a population size of

TABLE I
RULE BASE OF THE TWO FLCS

e \ ė	N	Z	P
N	NB	NS	Z
Z	NS	Z	PS
P	Z	PS	PB

200 chromosomes coded in real number. Members of the first generation are randomly initialized and the GA terminates after 50 generations. To ensure that the fitness increases monotonically, the best population in each generation enters the next generation directly. In addition, a generation gap of 0.8 is used during the reproduction operation so that 80% of the members in the new generation are determined by the rank reproduction method, while the remaining 20% are selected randomly. This strategy helps to prevent premature convergence. The crossover rate is 0.8. Mutation is performed by generating a random number, m, for each gene in the chromosome. If m is smaller than 0.05, non-linear mutation, as defined in Equation (6), occurs.

$$x(i+1) = x(i) + \delta(x, i) \tag{6}$$

where

$$\delta(x,i) = \begin{cases} (R_x - x)\cdot(1-\lambda^{(1-\frac{i}{1+i_{max}})}), & \text{if } rand(1) > 0.5 \\ (L_x - x)\cdot(1-\lambda^{(1-\frac{i}{1+i_{max}})}), & \text{otherwise} \end{cases} \tag{7}$$

and $x(i)$ is the value of gene x in i^{th} generation, i_{max} is the maximum number of generations, $[L_x, R_x]$ is the interval in which x lies, λ is a random number in [0, 1]. This mutation method enables finer adjustment to occur as i becomes bigger.

The fitness of each member in the population is assessed using the simulated liquid level process described in Section III. As the main objective of the control system is to minimize the error between the setpoint and the actual response of the plant, the fitness function is chosen as the sum of the integral of time-weighted absolute error (ITAE), which is defined in Equation (8).

$$F = \sum_{i=1}^{M}\left[\sum_{k=1}^{N_i} k * e(k)\right] \tag{8}$$

where N_i is the number of sampling instants between step changes in the setpoint, M is the number of step changes in the test sequence used to evaluate the fitness of each member in a generation. A test sequence comprising of multiple step changes is needed because the liquid level process has nonlinear dynamics. However, a complex setpoint trajectory places a higher computational load on the computer and it will take a longer time to evolve the necessary parameters. As a trade-off, M is chosen to be 2. The 2 setpoints are randomly selected at the start of each generation in order to cover a wide operating range. In addition, the process is first brought to the higher setpoint and then to the lower one, as illustrated in Fig. 10. This ensures that MFs that cover the both positive and negative sides of universe of discourse can be trained.

B. Type-1 FLC (FLC_1)

Since each universe of discourse of FLC_1 are partitioned by 3 fuzzy sets that have partition of unity, three points are sufficient to uniquely determine the three input MFs associated with an input domain. Using the MFs of e as an example, the three points are N_e, Z_e and P_e, as indicated by the dotted lines in Fig. 6. The output has five type-1 fuzzy sets. They may be represented mathematically by five distinct numbers because height defuzzifier is used. Consequently, there are a total of 11 parameters which need to be optimized by the GA. The MFs of e and $\dot{e}$ of FLC_1 evolved by GA are shown in Fig. 7 as the dark thick lines. The MFs of $\dot{u}$ are shown in Fig. 8.

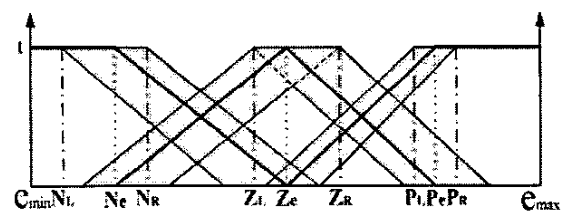


Fig. 6. Example membership functions of e

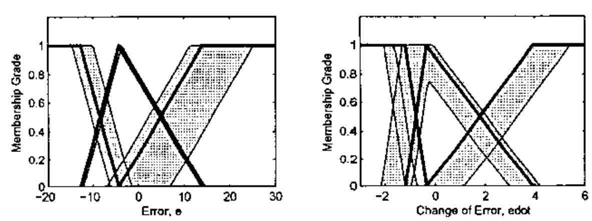


Fig. 7. MFs of e and $\dot{e}$

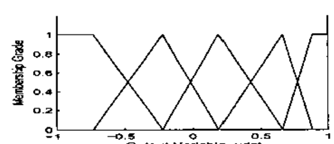


Fig. 8. MFs of $\dot{u}$

C. Type-2 FLC (FLC_2)

FLC_2 is a singleton interval type-2 FLC. It has the same fuzzy rule base and the output MFs as the type-1 FLC. In line with the partially dependent approach, the type-2 fuzzy input sets are designed by blurring the type-1 fuzzy sets described in the previous section. As illustrated in Fig. 2, the type-2 fuzzy sets may be generated by shifting the apex of the type-1 fuzzy set to the left and to the right. In other words, the FOU may be uniquely defined by selecting suitable values for the two dotted lines labelled as L and R.

GA is used to optimize the FOU by choosing the 12 parameters that define the FOU of the 6 type-2 input fuzzy sets in the antecedent part of the fuzzy "IF-THEN" rules. The coding strategy of the entire chromosome is illustrated in Fig. 9(a), and that for the sub-chromosome of e is illustrated in Fig. 9(b). The definitions of the genes are labelled in Fig. 6. In a bid to balance the need to reduce the search space and the need to provide sufficient flexibility, the range in which the parameters associated with the **N**, **Z** and **P** fuzzy sets of e can lie is first initialized as

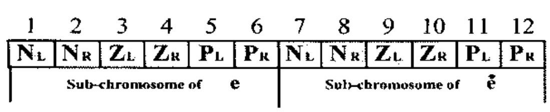
$$\begin{aligned} N_L,\ N_R &\in [e_{min}, (N_e + Z_e)/2] \\ Z_L,\ Z_R &\in [(N_e + Z_e)/2,\ (P_e + Z_e)/2] \\ P_L,\ P_R &\in [(P_e + Z_e)/2,\ e_{max}] \end{aligned}$$

At the end of each generation, the intervals are modified according to the best parameters obtained. The new intervals are

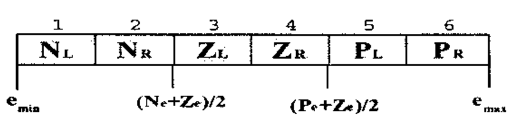
$$\begin{aligned} N_L,\ N_R &\in [e_{min}, (N_{RB} + Z_{LB})/2] \\ Z_L,\ Z_R &\in [(N_{RB} + Z_{LB})/2,\ (Z_{RB} + P_{LB})/2] \\ P_L,\ P_R &\in [(Z_{RB} + P_{LB})/2,\ e_{max}] \end{aligned}$$

where N_{RB}, Z_{LB}, Z_{RB}, P_{LB} are the parameters corresponding to the best chromosome obtained thus far.

A flexible position-coding strategy is also applied in each interval to improve the diversity of the members in each generation. Consequently, the order of the genes in the sub-chromosome may not remain in the same order, i.e. the gene corresponding to N_L may be larger than N_R. Each time after the chromosome has been changed, it is re-arranged in the right order.



(a) The complete chromosome



(b) The sub-chromosome of e

Fig. 9. Coding scheme for FOU of type-2 fuzzy set

The type-2 MFs evolved by the above GA for FLC_2 are shown in Fig. 7 as the shaded region. The MFs of $\dot{u}$ are the same as shown in Fig. 8. Note that the first MF for the type-1 fuzzy set for $\dot{e}$ domain does not lie within the FOU of its corresponding type-2 fuzzy set. This is a result of the strategy of adapting the range in which the FOU can lie.

V. Experimental Comparative Study

This section presents the results from the experimental study that was conducted on the actual plant to assess the performance of the type-1 and type-2 FLCs evolved by GA. The volumetric flow rate of the pumps in the coupled-tank apparatus used to produce the results is nonlinear and the system has non-zero transport delay [12]. These characteristics were not modelled by the simulated liquid level process used by the GA to optimize the fuzzy controller parameters. Hence, the ability of the FLCs to handle modelling uncertainties can be ascertained by examining control performance of the FLCs on the actual plant.

The responses to step changes in the setpoints and the corresponding control signals are shown in Fig. 10. The control performances are comparable to those obtained using a neuro-fuzzy controller [12], indicating that both FLCs are able to provide satisfactory control performance in the presence of uncertainties introduced by the pump non-linearity and the unmodelled transport delay.

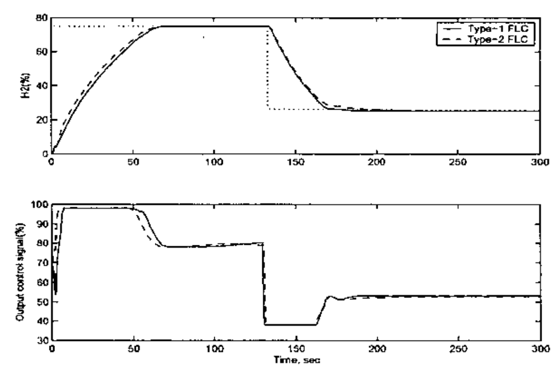


Fig. 10. Step responses to changed setpoints

To further compare the ability of the type-1 and type-2 FLCs to handle uncertainties, the flow rate between the two tanks was reduced by lowering the baffle separating the two tanks. This change gave rise to a system with slower dynamics. In addition, the difference in liquid level between the two tanks was larger at steady state. When the baffle was slightly lowered, the step responses and the control signal are shown in Fig. 11. When the baffle was further lowered, the results are shown in Fig. 12. It may be observed that both the FLCs are able to attenuate the oscillations when the modelling uncertainties are small. The liquid level in the tank will eventually reach the desired setpoint, though the settling time is shorter when FLC_2 is employed. When the modelling uncertainties are bigger, FLC_1 will give rise to persistent oscillations while FLC_2 has the ability to eliminate these oscillations and the liquid level reach its desired height at steady state.

Lastly, the ability of the two FLCs to deal with a larger unmodelled transport delay was studied. First, a 2-second transport delay was added to the system. The step responses and the control signal are shown in Fig. 13. When a 4-second transport delay was added to the system, the results are shown in Fig. 14. Once again, the type-2 FLC outperforms its type-1 counterpart especially when the uncertainty is large. Thus, it may be concluded that the type-2 FLC is more robust than the type-1 FLC.

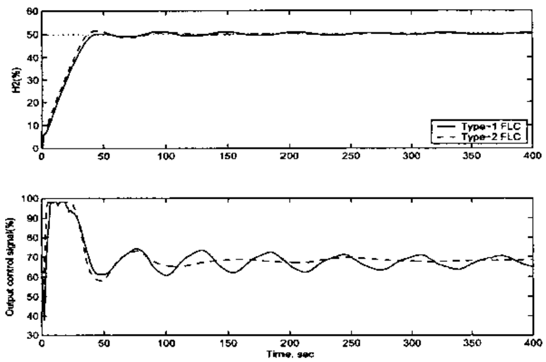


Fig. 11. Step responses when the baffle was slightly lowered

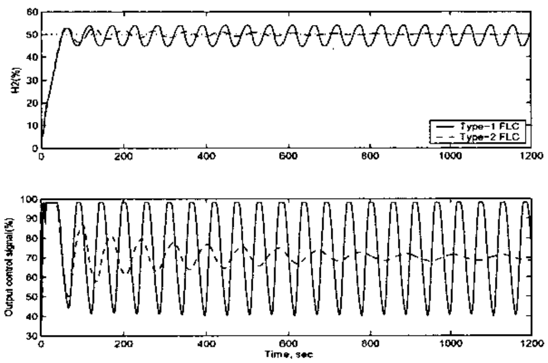


Fig. 12. Step responses when the baffle was further lowered

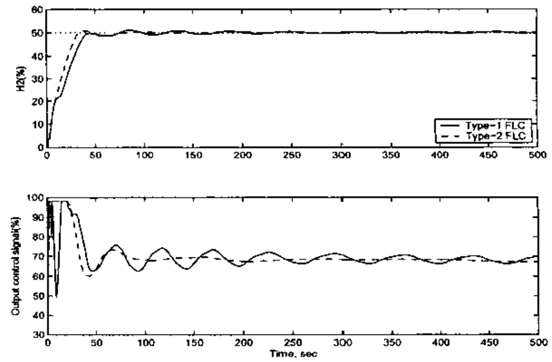


Fig. 13. Step responses when there was a 2 sec transport delay

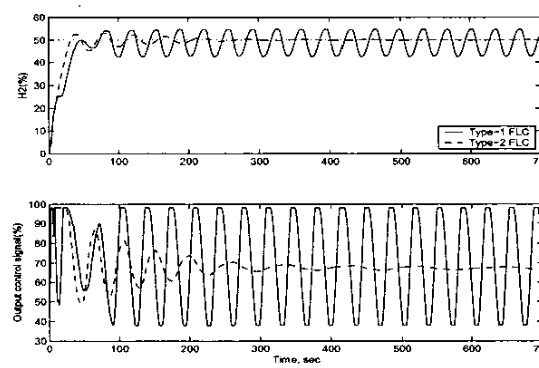


Fig. 14. Step responses when there was a 4 sec transport delay

VI. Discussions

Besides control performance, another issue to consider is the amount of computing power needed to implement a type-2 FLC. Comparatively, the computational cost of FLC_2 is higher. On a 500 MHz computer with 128M RAM, the average computation time for FLC_1 and FLC_2 are 9.5 milliseconds and 16.8 milliseconds respectively. As the number of MFs, and therefore the rule base, become larger, the computational cost will increase further. Nevertheless, the larger computational requirements is not likely be a serious problem as fast computers are readily available nowadays. The computational load can also be reduced by employed the uncertainty bound type-reduction method [11].

VII. Conclusions

In this paper, a GA approach for designing a singleton interval type-2 FLC is proposed and used to design a controller for a coupled-tank liquid-level system. Experimental results show that the type-2 FLC outperforms its type-1 counterpart significantly when the modelling error is large. The main advantage of the type-2 FLC appears to be its ability to eliminate persistent oscillations. As the type-2 FLC can tolerate bigger modelling errors, it is more robust than type-1 FLCs. Another interesting observation is that the performances of both FLCs are similar for the nominal plant, indicating that robustness is obtained with little performance trade-offs.

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