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A Simplified Architecture for Type-2 FLSs and Its Application to Nonlinear Control

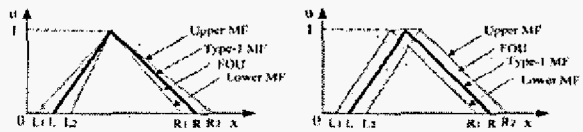
Dongrui Wu
Department of Electrical and Computer Engineering
National University of Singapore
Singapore 117576
E-mail: g0305786@nus.edu.sg

Woei Wan Tan
Department of Electrical and Computer Engineering
National University of Singapore
Singapore 117576
E-mail: eletanww@nus.edu.sg

***Abstract*— A type-2 fuzzy logic system (FLS) is one that has at least one type-2 membership function (MF) in its rule base. Consequently, the output of the inference engine is a type-2 fuzzy set and must be type-reduced before the defuzzifier is able to convert the output set into a crisp value. Type-2 FLS may not be suitable for certain real-time applications because type-reduction is very computationally intensive, especially when there are many MFs and the rule base is large. In this paper a simplified architecture for type-2 FLSs is proposed, where only one fuzzy set for each input domain is type-2 and all others are type-1. This architecture relieves the computational burden of the type-2 fuzzy system, while preserving its advantages over traditional type-1 FLSs. Two FLSs that have the proposed architecture are used to control a nonlinear SISO plant. Experimental results show that they cope well with the complexity of the plant, and can handle the modelling uncertainties better than their type-1 counterpart.**

I. Introduction

Type-1 fuzzy logic was first introduced by Zadeh in 1965 [1]. Fuzzy logic systems (FLSs) constructed based on type-1 fuzzy sets, referred to as type-1 FLSs, have demonstrated their ability in many applications [2], especially for the control of complex nonlinear systems that are difficult to model analytically [3], [4]. However, researches have shown that type-1 FLSs have difficulties in modelling and minimizing the effect of uncertainties [5]. A reason being that a type-1 fuzzy set is certain in the sense that for each input, there is a crisp membership grade. Type-2 fuzzy sets, characterized by membership grades that are themselves fuzzy, were introduced by Zadeh in 1975 [6] to account for this problem. As illustrated in Fig. 1, the membership function of a type-2 set have a footprint of uncertainty (FOU), which represents the uncertainties in the shape and position of the type-1 fuzzy set. The FOU is bounded by an upper MF and a lower MF, both of which are type-1 MFs. FLSs constructed using rule bases that utilises at least one type-2 fuzzy sets are called type-2 FLSs. Since the FOU of a type-2 fuzzy set provides an extra mathematical dimension, they are very useful in circumstances where it is difficult to determine an exact membership grade for a fuzzy set. Hence, type-2 FLSs can be used to handle system uncertainties and have the potential to outperform their type-1 counterparts. The structure of a type-2 FLS is shown in Fig. 2. Compared with a type-1 FLS, the main difference is that a type-reducer is needed to convert the type-2 fuzzy output sets into type-1 sets so that they can be processed by the defuzzifier to give a crisp output.



(a) A type-2 fuzzy set evolved by blurring the width of a type-1 fuzzy set

(b) A type-2 fuzzy set evolved by blurring the apex of a type-1 fuzzy set

Fig. 1. Type-2 fuzzy sets

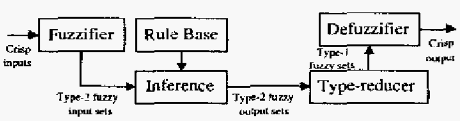


Fig. 2. A type-2 fuzzy logic system

Previous researches [7], [8] indicate that the ability of type-2 FLCs to eliminate persistent oscillations surpass that of their type-1 counterparts. Reason being that the control surface of a type-2 FLC is smoother than that of a type-1 FLC, especially around the origin. As a result, small disturbances around steady state will not result in significant control signal changes and thus minimising the amount of oscillations. The additional mathematical dimension provided by the FOU allows a type-2 FLC to handle modelling uncertainties better than conventional type-1 FLCs. This advantage is particularly useful because many fuzzy controllers are designed offline using genetic algorithm (GA) and a model of the controlled process. As it is impossible for a model to capture all the characteristics of the actual plant, the performance of a controller designed using a model will inevitably deteriorate when it is applied on the actual plant. A controller that is equipped with the ability to handle modelling uncertainties would be valuable. Hence, a promising strategy is to use GA to evolve type-2 FLCs, instead of type-1 FLCs.

Despite the advantages offered by type-2 FLS, one problem that may hinder the use of type-2 FLCs for real-time control is high computational cost. Type-reduction is very computationally intensive, especially when there are many MFs and the rule base is large. To reduce the computational burden while preserving the advantages of type-2 FLCs, two approaches may be considered : 1) faster type-reduction methods, such as the uncertainty bound concept in [9]; and 2) a simpler architecture. The second approach is adopted herein. A simplified architecture for type-2 FLCs is proposed, where only one fuzzy set in each input domain is type-2 and all others are type-1. The structure is motivated by the observation that the main advantage of type-2 FLC appears to be its ability to provide more damping as the output approaches the set-point. It is conjectured that the degradation in the ability of a type-2 FLC to handle modelling uncertainties will be insignificant if type-1 fuzzy sets are used to describe the fuzzy rules that govern the transient response. This paper presents the experimental study that was conducted to assess the ability of the simplified type-2 structure to cope with modelling uncertainties.

The rest of the paper is organized as follows: Section II introduces the interval singleton type-2 FLS. Section III proposes the simplified architecture of type-2 FLSs and describes how GA is used to tune the controller parameters. The computational load of the proposed structure is then compared with other types of FLCs in Section IV. Results of a study to compare the ability of four FLCs, one type-1 and three type-2, to handle modelling uncertainties are presented in Section V. Section VI discusses the performances of the proposed architecture. Finally, conclusions are drawn in Section VII.

II. Interval Singleton type-2 FLS

In this section, an interval singleton type-2 FLS [5] is introduced. The term "interval" means that only interval type-2 fuzzy sets, where all points in the FOU have unity secondary membership grades, are utilised. "Singleton" denotes that the fuzzifier converts the inputs signals of the FLC into fuzzy singletons.

A. Fuzzy Inference Engine

For an interval singleton type-2 FLS, the inference engine combines the fuzzy rules in order to map the crisp inputs to interval type-2 fuzzy output sets. Just as the sup-star composition is the backbone of a type-1 FLS, the extended sup-star composition is the backbone for a type-2 FLS. To illustrate the extended sup-star operation, consider a rule base that consists of the following rules :

$$R^{ij} : \text{If } \widetilde{F}_1^i \text{ and } \widetilde{F}_2^j \rightarrow \widetilde{G}^{ij}$$

where $i, j = 1, 2, \ldots, N$ and N is the number of MFs of each input.

Since interval type-2 fuzzy sets are used, the firing set $F^{ij}(\mathbf{x})$ obtained when the union operation is implemented by the mathematical product is the interval type-1 set :

$$F^{ij}(\mathbf{x}) = \mu_{\widetilde{F}_1^i}(x_1) \cdot \mu_{\widetilde{F}_2^j}(x_2) = [\underline{f}^{ij}(\mathbf{x}), \overline{f}^{ij}(\mathbf{x})] \equiv [\underline{f}^{ij}, \overline{f}^{ij}] \quad (1)$$

where $\underline{f}^{ij}(\mathbf{x}) = \underline{\mu}_{\widetilde{F}_1^i}(x_1) \times \underline{\mu}_{\widetilde{F}_2^j}(x_2)$ and $\overline{f}^{ij}(\mathbf{x}) = \overline{\mu}_{\widetilde{F}_1^i}(x_1) \times \overline{\mu}_{\widetilde{F}_2^j}(x_2)$. $\underline{\mu}_{\widetilde{F}^i}(x)$ and $\overline{\mu}_{\widetilde{F}^j}(x)$ are the lower and upper membership grades of $\mu_{\widetilde{F}^i}(x)$. Equation (1) shows that the firing level of rule R^{ij} can be determined by computing the lower and upper membership grades. Type-reduction and defuzzification are then performed to calculate the crisp output.

B. Type-Reduction and Defuzzification

The output corresponding to the fired rule is a type-2 fuzzy set which must be type-reduced before the defuzzifier can be used to generate a crisp output. This is the main structural difference between type-1 and type-2 FLSs. In this paper, the center-of-sets type-reducer is used. It combines all the type-2 output sets and then performs a center-of-sets calculation to produce a type-1 set, known as a type-reduced set. The Karnik-Mendel iterative type-reduction method [5], which is adopted in this work, is based on the Generalized Centroid(GC) concept [5].

$$\begin{aligned} GC &= \int_{z_1 \in Z_1} \cdots \int_{z_N \in Z_N} \int_{w_1 \in W_1} \cdots \int_{w_N \in W_N} 1 \Bigg/ \frac{\sum_{i=1}^{N} z_i w_i}{\sum_{i=1}^{N} w_i} \\ &= [y_l, \ y_r] \end{aligned} \quad (2)$$

where W_i is a type-1 set with center h_i and spread δ_i that represents the firing level of rule R^i. When interval type-2 sets are used, then W_i assumes the expression defined in Equation (1). Depending on the defuzzification method employed, each Z_i can be a type-1 fuzzy set or a crisp number. For example, Z_i is a type-1 set if centroid defuzzification is used. As height defuzzification is employed in this paper, Z_i is a crisp number. Once y_l and y_r are obtained via the Karnik-Mendel iterative type-reduction method, the type-reduced set can be defuzzified to calculate the crisp output. For an interval type-reduced set, the defuzzified output is $\frac{y_l + y_r}{2}$.

III. FLCs Design Strategies

In this section, the GA-based strategy that may be employed to tune the parameters of FLCs are described. The partially dependent approach [8] for designing type-2 FLS was utilised. A baseline type-1 FLC is first designed. Type-2 FLCs may then be evolved by either blurring the width or the centres of the type-1 fuzzy sets (See Fig. 2).

A. Structure of the FLCs

To provide a common basis for comparison, all the FLCs have essentially the same architecture. The only difference being that the input domains of the type-1 FLC are partitioned by type-1 sets, while that of the type-2 FLCs are partitioned by at least one type-2 set. The feedback error, e, and the change of the error, $\dot{e}$, are the input signals of the FLCs and the output signal is the change in the control signal, $\dot{u}$. Each input domain is partitioned by three fuzzy MFs that are labelled as **N**, **Z** and **P**. The output space has five MFs labelled as **NB**,

NS, **Z**, **PS** and **PB**. The various fuzzy set operations adopted in this paper are the sum-product inference engine, center-of-sets type-reducer and height defuzzifier. Table I shows the fuzzy rule base used by the four FLCs. It is commonly used to construct FLCs.

$e \backslash \dot{e}$	$N_{\dot{e}}$	$Z_{\dot{e}}$	$P_{\dot{e}}$
N_e	NB	NS	Z
Z_e	NS	Z	PS
P_e	Z	PS	PB

TABLE I

RULE BASE OF THE THREE FLCS

B. The Type-1 FLC

As the input domain of the type-1 FLC (FLC_1) is partitioned by three MFs, three points are needed to determine the MFs of each input. As illustrated in Fig. 3, the three points for the e domain are N_e, Z_e and P_e. Another five points are needed to determine the MFs of $\dot{u}$. Consequently, there is a total of 11 parameters which need to be optimized by the GA. The GA coding scheme of FLC_1 is illustrated in Fig. 4 as the first 11 genes.

C. Type-2 FLCs

The main difference between the control surfaces of type-1 and type-2 FLCs is the area around the origin [7], [8]. This observation indicates that the fuzzy sets characterising the region around the zero point (Z) play the most important role in improving the control performance of a type-2 FLC. Thus, the proposed simplified architecture is one where only Z is a type-2 fuzzy set and all remaining fuzzy sets are type-1 (Refer to Fig. 3). Such a simplified structure and a FLC where all the fuzzy sets used to partition the input domains are type-2 may have similar control surfaces around the origin. If the control surfaces are comparable, these two kinds of FLCs would likely have similar performances. Since the simplified architecture utilises fewer type-2 sets, a reduction in computational cost can be achieved without sacrificing performance.

Type-2 FLCs evolved by GA are used to test the hypothesis that the simplified architecture retains the ability to handle modelling uncertainties. As illustrated in Fig. 1, a type-2 fuzzy set can be obtained by blurring the membership function of a baseline type-1 set. For a triangular type-1 MF, there are at least two ways of blurring to obtain a type-2 MF. The first is to keep the apex fixed while blurring the width of the triangle, as shown in Fig. 1(a). The other way is to keep the width of the triangle fixed while blurring the apex, as shown in Fig. 1(b). Both methods for constructing a type-2 set are employed in this paper. The type-2 FLC designed by blurring the width of the fuzzy set Z is denoted FLC_{2c}, the other type-2 FLC designed by blurring the apex of the fuzzy set is denoted FLC_{2d}.

For simplicity, the amount by which the end-points of the type-1 fuzzy set is shifted to the left is equal to the rightward shift. The distances $L_1L = LL_2 = R_1R = RR_2$ in the e domain can, therefore, be denoted by a single parameter de (Refer to Fig. 3). Likewise, the leftward and rightward shift needed to construct the type-2 fuzzy set for the $\dot{e}$ domain is $d\dot{e}$. As only one parameter is sufficient to determine the FOU of the type-2 fuzzy set in each input domain, the chromosome used to evolve the controller parameters of FLC_{2c} and FLC_{2d} has 13 genes. The first 11 genes are the same as those in the chromosome of the type-1 FLC. The last two genes are used to determine the amount of shift to generate the FOU of the type-2 fuzzy set used to partition the e and $\dot{e}$ domain.

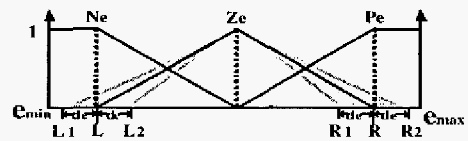


Fig. 3. Example membership functions of e

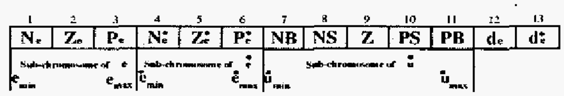


Fig. 4. GA coding scheme of the FLCs

In order to study whether the proposed structure can preserve the advantages of traditional type-2 FLCs, another type-2 FLC (FLC_{2f}) is needed for comparison. Its structure is similar to FLC_{2d}. However, all of its MFs are type-2. In addition, the restriction that $L_1L = R_1R$ is lifted so that FLC_{2f} has much more design degree of freedom than FLC_{2c} and FLC_{2d}.

IV. COMPUTATIONAL COST

The main advantage of the simplified structure is lower computation cost. Hence, it would be useful to ascertain the amount of reduction in computing power needed to implement the various FLCs.

Without loss of generality, assume that N equally spaced MFs are used to partition each of the two $[-1, 1]$ input domains. The FOU of every type-2 MF is defined as $d_e = d_{\dot{e}} = \frac{1}{N-1}$, i.e. half of the distance between two adjacent apexes. This study is conducted by first generating the 101 points, $e_i = 2(i-1)/100 - 1, i = 1, \ldots, 101$, that divide the error domain into 100 equally-spaced intervals. Another 101 points in the rate of change in error domain are generated in a similar manner. By combining these points in all possible ways, 10201 input vectors are generated. Computational cost is evaluated by comparing the time needed to calculate the outputs corresponding to these 10201 input vectors. The platform is a 1G Hz computer with 256M RAM and Windows XP running MATLAB. Employing the Karnik-Mendel type reducer, the computation time for the 4 FLCs are shown in Table II. Clearly, the computational time needed by FLC_{2c} and FLC_{2d} is much shorter than that of FLC_{2f}. In general, computation for the proposed structure is completed in half the time needed for a full type-2 FLC.

N \ FLC	FLC_1	FLC_{2c}	FLC_{2d}	FLC_{2f}
3	1.2 sec	6.3 sec	6.7 sec	10.4 sec
5	1.6 sec	4.6 sec	5.0 sec	10.2 sec
7	2.3 sec	4.7 sec	5.1 sec	12.1 sec
9	3.2 sec	6.1 sec	6.4 sec	15.0 sec
11	4.5 sec	8.9 sec	9.4 sec	19.7 sec

TABLE II

COMPARISON OF COMPUTATIONAL COST

Unlike FLC_1, the computation time for FLC_{2c}, FLC_{2d} and FLC_{2f} does not increase monotonically as N increases. There is a drop in computing time when N increases from 3 to 5. This may be because the FOU ($d_e = d_{\dot{e}} = \frac{1}{N-1}$) of FLC_{2c} and FLC_{2d} for the $N = 5$ case is smaller so the type-2 sets are fired less, and therefore the type reducer is activated fewer number of times. When N is increased further, the total number of firing strengths that need to be calculated increases so the amount of computing power required rises.

V. EXPERIMENTAL ASSESSMENT OF SIMPLIFIED TYPE-2 ARCHITECTURE

In this section, results from experiments conducted to examine whether a FLC constructed according to the proposed simplified architecture is able to handle modelling uncertainties are presented.

A. The Coupled-tank System

Fig. 5 shows the experimental setup. The equipment consists of two small tower-type tanks mounted above a reservoir that store the water. Water is pumped into the top of each tank by two independent pumps, and the levels of water are measured by two capacitive-type probe sensors. Each tank is fitted with an outlet at the side near the base. Raising the baffle between the two tanks allows water to flow between them. The amount of water which returns to the reservoir is approximately proportional to the square root of the height of water in the tank, which is a main source of nonlinearity.

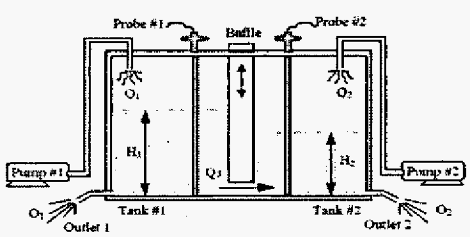


Fig. 5. The coupled-tank liquid-level control system

The dynamics of the coupled-tank apparatus can be modelled by the following set of nonlinear differential equations :

$$A_1\frac{dH_1}{dt} = Q_1 - \alpha_1\sqrt{H_1} - \alpha_3\sqrt{H_1 - H_2} \quad (3a)$$

$$A_2\frac{dH_2}{dt} = Q_2 - \alpha_2\sqrt{H_2} + \alpha_3\sqrt{H_1 - H_2} \quad (3b)$$

	I	II	III	IV
$A_1 = A_2$ (cm^2)	36.52	36.52	36.52	36.52
$\alpha_1 = \alpha_2$	5.6186	5.6186	5.6186	5.6186
α_3	10	10	10	8
Setpoint (cm)	$0 \rightarrow 15$	$0 \rightarrow 22.5 \rightarrow 7.5$	$0 \rightarrow 15$	$0 \rightarrow 15$
Transport delay (s)	0	0	2	0

TABLE III

PLANTS USED TO ASSESS FITNESS OF CANDIDATE SOLUTIONS

where A_1, A_2 are the cross-sectional area of Tank #1, #2; H_1, H_2 are the liquid level in Tank #1, #2; Q_1, Q_2 are the volumetric flow rate (cm^3/sec) of Pump #1, #2; $\alpha_1, \alpha_2, \alpha_3$ are the proportionality constant corresponding to the $\sqrt{H_1}$, $\sqrt{H_2}$ and $\sqrt{H_1 - H_2}$ terms.

The coupled-tank apparatus can be configured as a second-order SISO system by turning off Pump #2 and using Pump #1 to control the water level in Tank #2. FLCs were tuned based on the simulated plant. The sampling period is 1 sec. For the results reported herein, the following process parameters are adopted:

$$A_1 = A_2 = 36.52 \text{ cm}^2 \quad (4a)$$

$$\alpha_1 = \alpha_2 = 5.6186 \quad (4b)$$

$$\alpha_3 = 10 \quad (4c)$$

The available DC voltage supply is [0, 5] V. In this paper the maximum control signal used is 4.906 V, corresponding to an input flow rate of about 73 cm^3/sec. To compensate the dead zone, the minimum control signal is chosen to be 1.646 V. These parameters match the actual plant which was used to test the controllers.

B. GA Parameters

The simulation model of the liquid level process described in Equation (3) is used to generate step responses in order to assess the fitness of each chromosome in the GA population. The Integral of the Time Absolute Error (ITAE) is the basis upon which the control performances are judged. As this paper aims at to exploring the two FLCs' ability to handle modelling uncertainties, each candidate solution is used to control the four different plants shown in Table III.

The sum of the four ITAEs, as defined in Equation (5), corresponding to the four cases is used to evaluate the fitness of the FLCs.

$$F = \sum_{i=1}^{4} \alpha_i \left[\sum_{j=1}^{N_i} j * e_i(j) \right] \quad (5)$$

where α_i is the weight corresponding to the ITAE of the ith plant, and $N_i = 200$ is the number of sampling instants. There is a need to introduce α_i because the ITAE of the second plant is usually several times bigger than that of other plants. To ensure that the ITAE of the four plants can be reduced with equal emphasis, α_2 is defined as $\frac{1}{3}$ while the other weights are unity. The MFs of FLC_1, FLC_{2c}, FLC_{2d} and FLC_{2f} evolved by GA are shown in Fig. 6.

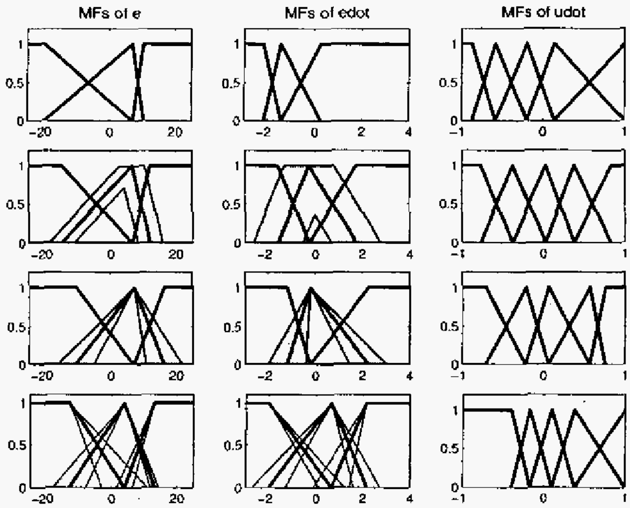
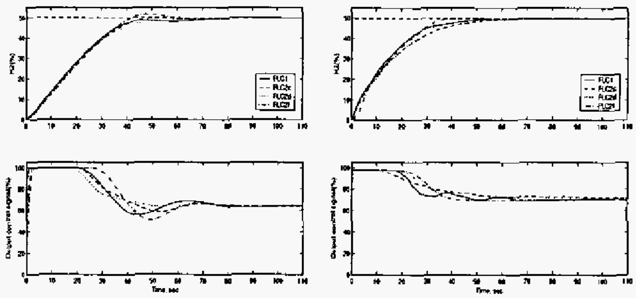
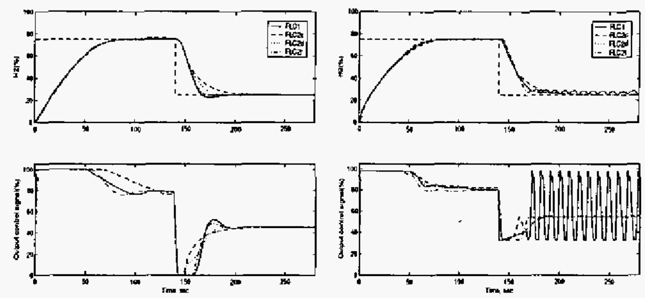


Fig. 6. MFs of the four FLCs



(a) Simulation results (b) Experiment results

Fig. 7. Step responses when the setpoint was 15 cm



(a) Simulation results (b) Experiment results

Fig. 8. Step responses when the setpoint was changed

C. Experimental Results

The results from the simulation and experimental study that was conducted to assess the performance of the type-1 and type-2 FLCs evolved by GA are presented here. As pointed out in [10], the volumetric flow rate of the pumps in the coupled-tank apparatus used to produce the results is nonlinear and the system has non-zero transport delay. These characteristics are not accurately captured by the plants used by the GA to optimize the fuzzy controller parameters. Hence, the ability of the four FLCs to handle modelling uncertainties can be ascertained by examining control performances of the FLCs.

The step responses obtained using FLC_1, FLC_{2c}, FLC_{2d} and FLC_{2f} with different setpoints and the corresponding control signal are shown in Fig. 7 and Fig. 8. It is found that when the setpoint changes from 22.5 cm to 7.5 cm, FLC_1 will result in persistent oscillations, though the simulation result is stable. This may suggest that the type-1 FLC is not so good at handling unmodelled dynamics. The control performances of the type-2 FLCs coincide with the simulation results and are comparable to those obtained using a neurofuzzy controller reported in [10], indicating that they can handle the uncertainties introduced by the pump non-linearity and the unmodelled transport delay.

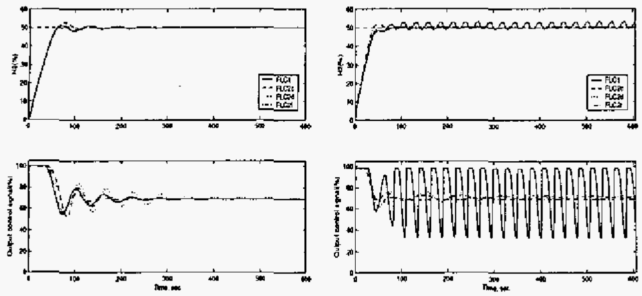
To further test the FLCs, the flow rate between the two tanks was reduced by lowering the baffle separating the two tanks. This change gave rise to a system with slower dynamics. In addition, the difference in liquid level between the two tanks was larger at steady state. When the baffle was lowered, the step responses and the control signal are shown in Fig. 9. It may be observed that although the simulation results of the four FLCs are similar, FLC_1 will give rise to persistent oscillations when tested on the actual plant, while the type-2 FLCs have the ability to eliminate these oscillations and the liquid level reaches its desired height at steady state.

Lastly, the ability of the FLCs to deal with transport delay was studied, as shown in Fig. 10 and Fig. 11. Once again, all type-2 FLCs outperform their type-1 counterpart. Thus, it may be concluded that all type-2 FLCs are more robust than the type-1 FLC.

VI. Discussions

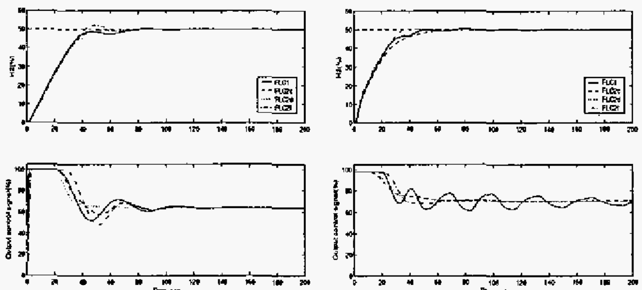
From the plots presented in Fig. 7-11, it may be concluded that the simulation and experimental results obtained using the type-2 FLCs generally coincide more than that of the type-1 FLC. In fact five type-1 FLCs were optimized by GA and tested on the practical plant. Most of them performed poorly. The responses either had long settling time or exhibited persistent oscillations. FLC_1 presented in the previous section is the best one chosen from these type-1 FLCs. Several type-2 FLCs from different runs were also tested on the actual plant. The experimental results did not differ significantly from the simulation results. The trait is indicative of the superior ability of type-2 FLCs to tolerate more modelling uncertainties. When a simulation model is used to evaluate the GA candidate solutions, the type-2 FLCs will have a higher probability of performing well on the actual plant.

Fig. 12 show the control surfaces of the four FLCs. The control surfaces of the three type-2 FLCs are smoother than that of the type-1 FLC, especially around the origin ($e = 0$, $\dot{e} = 0$). The smoother control surface demonstrates that the proposed architecture is superior to type-1 FLSs. The similarity between the three type-2 FLCs also show that the proposed structure will preserve the main advantages of type-2 FLCs.



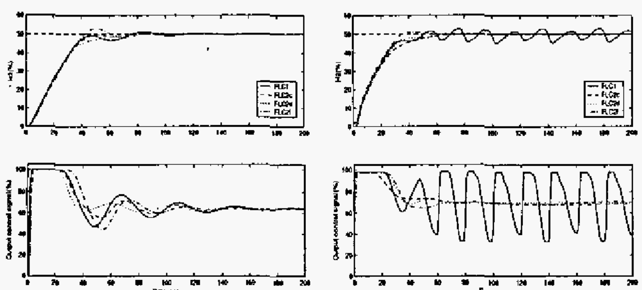
(a) Simulation results (b) Experiment results

Fig. 9. Step responses when the baffle was lowered



(a) Simulation results (b) Experiment results

Fig. 10. Step responses when there was a 1 sec transport delay



(a) Simulation results (b) Experiment results

Fig. 11. Step responses when there was a 2 sec transport delay

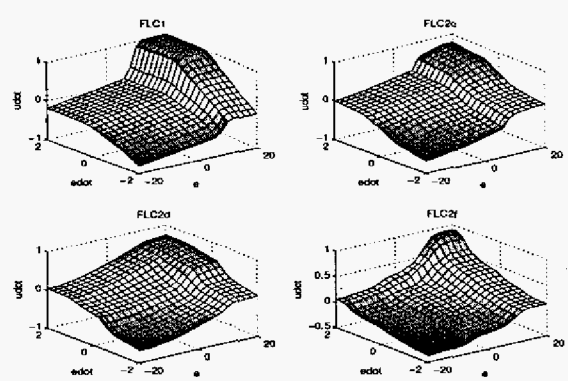


Fig. 12. Control surface of the four FLCs

In fact, a FLS with the proposed architecture has two parts—a type-1 part and a type-2 part. Different portions will be activated when the state of the plant is in different stage. When the state of the plant is far from the steady state, the type-2 FLC will behave like a type-1 FLC since no type-2 MFs are fired. Thus the plant is roughly controlled by the type-1 part. When the state approaches the steady state, type-2 MFs will be fired and the plant is fine controlled by the type-2 part. Smoother control signal will be generated, which helps to eliminate oscillations.

In this paper only 3 MFs are used to partition each input domain and only the middle one is chosen to be type-2. However, if more MFs are used for each input domain, more than one MFs may be chosen as type-2 depending on the need to smoothen the control surface. Thus, the proposed architecture provides more freedom in design by finding a balance between performance and computational cost. Since the MFs near two ends of the universe of each input are usually not useful near steady state, they can be chosen as type-1 sets. However, more analysis is needed to determine the optimal number type-2 MFs to use around the origin.

VII. Conclusions

A simplified architecture for type-2 FLSs has been introduced and two FLCs with this proposed architecture are designed for a coupled-tank liquid level control process. Their performances are compared with a type-1 FLC. Both the simulation and experimental results show that the type-2 FLCs outperform the type-1 one. The FLSs with the architecture proposed in this paper preserve the main advantages of traditional type-2 FLSs while lowering the computational burden at the same time. The simplified structure will be especially useful when the rule base is large and there are many MFs.

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